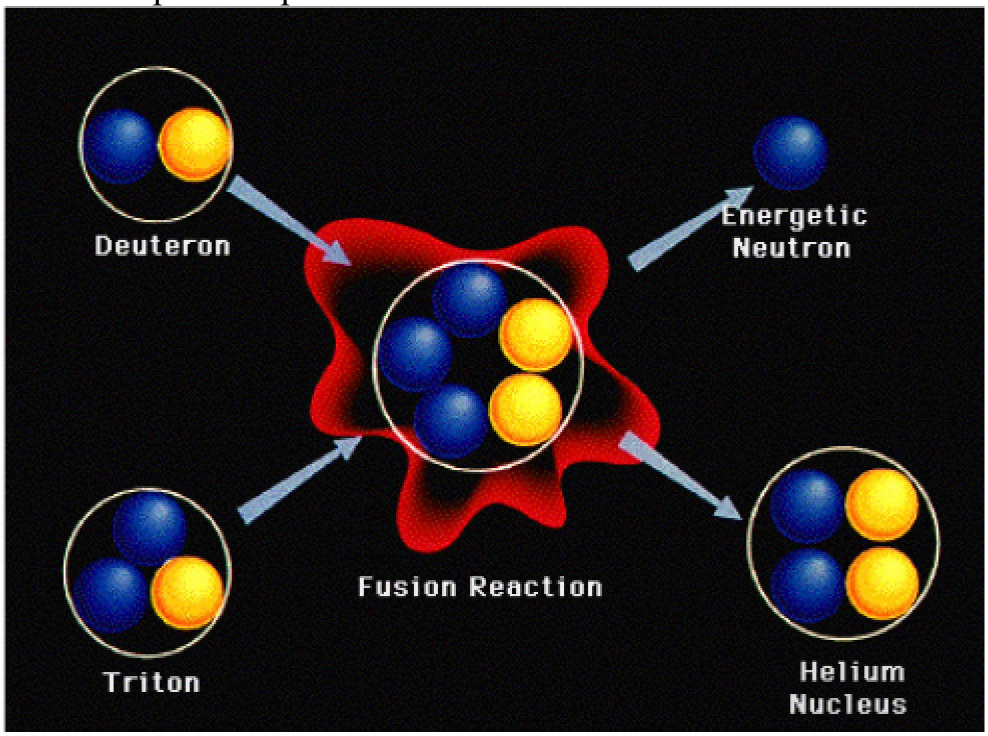
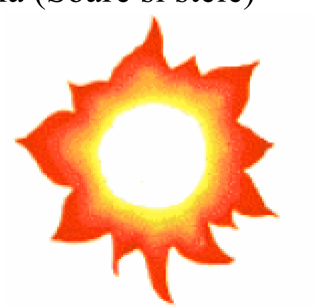
Cum este produsa plasma ?

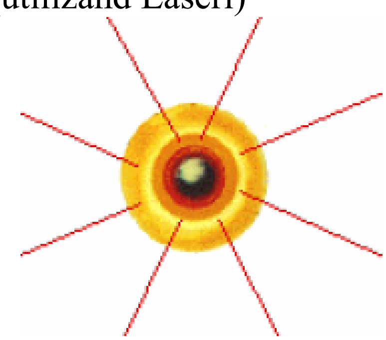


Cum poate fi confinata plasma?

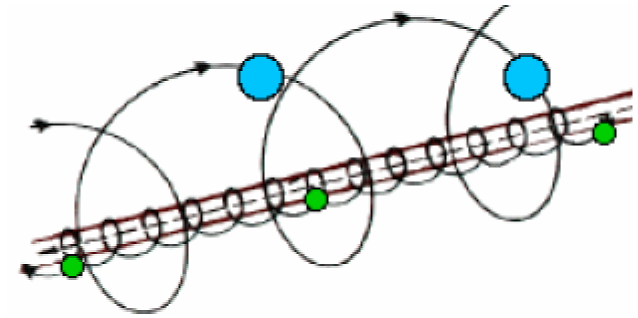
1) Confinare gravitationala (Soare si stele)



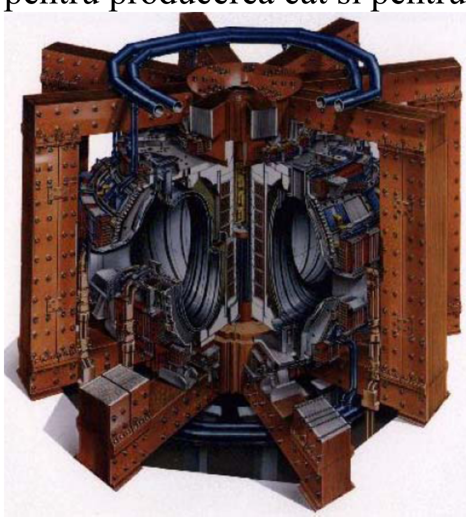
2) Confinare inertiala (utilizand Laseri)

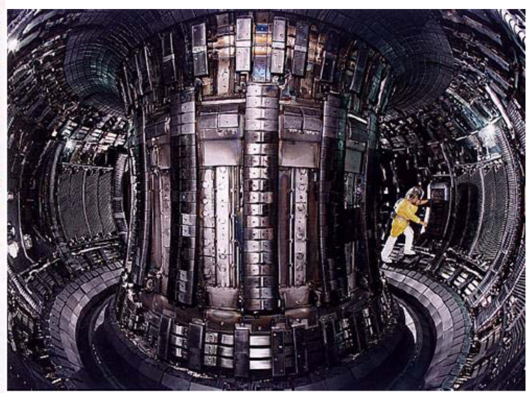


3) Confinare magnetica



Instalatii Tokamak: Liniile de magnetice de forta , produse de bobine solenoidale magnetice, sunt inchise sub forma unei “gogosi” protejand plasma si eliminand pierderile. Ele trebuie sa aiba o structura elicoidala pentru a stabiliza plasma. Aceasta deformare in structura lor geometrica se poate creea prin inducerea unui curent toroidal, curent ce poate fi utilizat atat pentru producerea cat si pentru incalzirea plasmei.

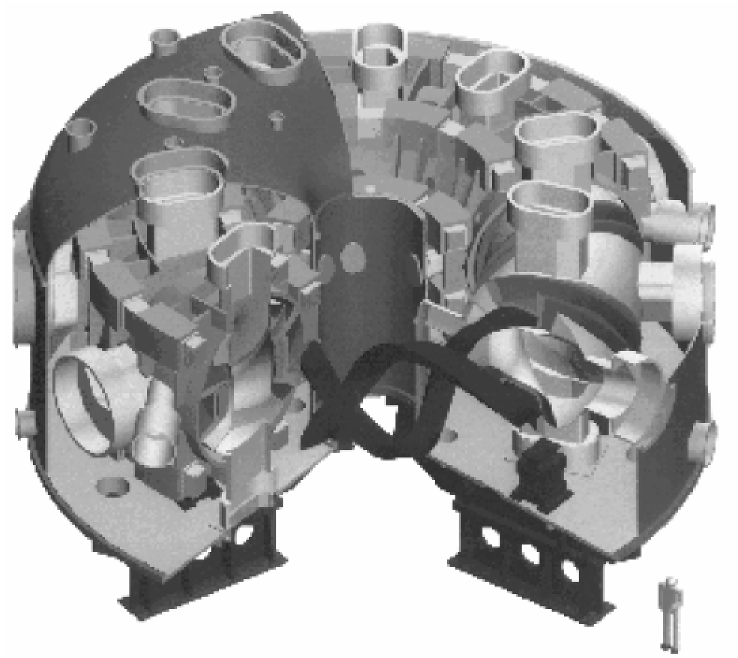




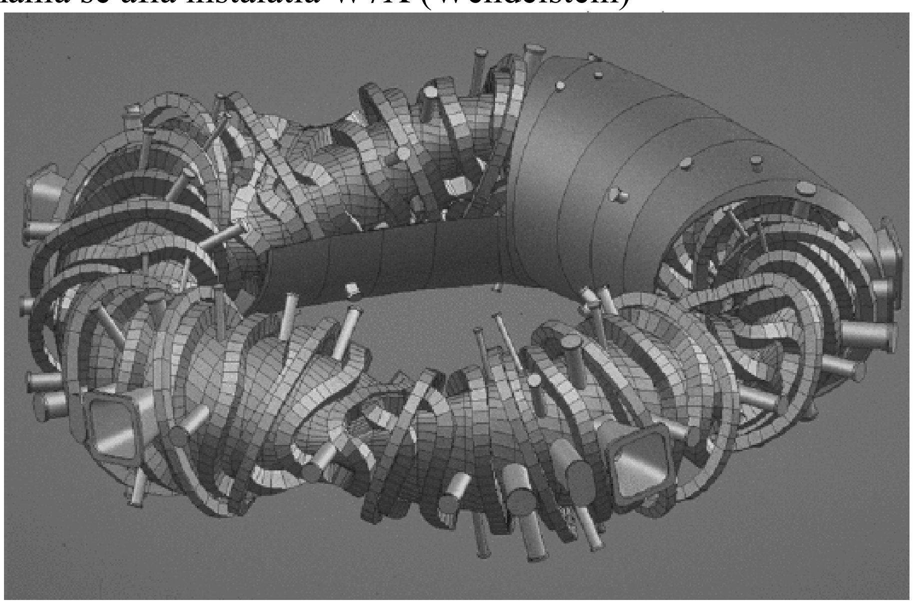
Cea mai mare instalatie Tokamak (Joint European Torus) din lume se afla la Culham (UK)

Instalatii Stellarator: Un alt mod de confinare magnetica prin care se realizeaza atat rasucirea elicoidala cat si cea toroidala a liniilor de camp magnetic, cu ajutorul unui magnet exterior plasmei.

In Japonia este cel ai mare Stellarator cu magneti supraconductori (LHD= Large Helical Device), numit heliotron

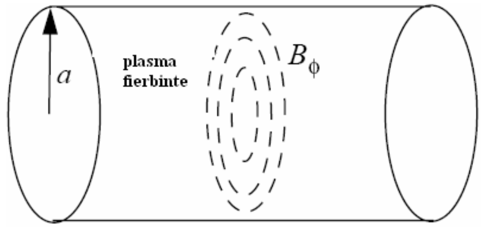


In Germania se afla instalatia W7X (Wendelstein)



Pinch-ul magnetic

Confinarea plasmei de catre un camp magnetic toroidal este un exemplu foarte bun de exemplificare a diferitelor forte generate de campul magnetic.



In acest caz, fortele de curbura associate cu"tensiuna" magnetica pot sa asigure confinarea plasmelor fierbinti. Aceasta posibilitate, poate fi demonstrata analizand configuratia de echilibru folosind ecuatia de miscare;

$$\rho \frac{d\vec{u}}{dt} = -\nabla p + \frac{1}{\mu}\left(\nabla \times \vec{B}\right) \times \vec{B} = 0$$

Introducem coordonatele polare cilindrice:

$$\vec{B} = \left(B_r, B_\phi, B_z\right) = \left(0, B_\phi, 0\right)$$

Astfel

$$\nabla \times \vec{B} = \left[\frac{1}{r}\frac{\partial B_z}{\partial \phi} - \frac{\partial B_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rB_\phi\right) - \frac{\partial B_r}{\partial \phi}\right]\hat{z} = -\frac{\partial B_\phi}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\hat{z}$$

$$\left(\nabla \times \vec{B}\right) \times \vec{B} = -\frac{B_\phi}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\hat{r} - B_\phi \frac{\partial B_\phi}{\partial z}\hat{z}$$

Consideram ca B_Φ este independent de z si

$$\left(\nabla \times \vec{B}\right) \times \vec{B} = -\frac{B_\phi}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\hat{r}$$

Echilibrul magnetostatic radial:

Limitarile impuse conduc la luarea in considerare, doar a componentei radiale a fortei de balans

$$-\frac{\partial p}{\partial r} - \frac{1}{\mu}\frac{B_\phi}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = 0$$

Relatie care pune in evidenta existenta unei mari varietati de tipuri de echilibre magnetostatice. Pentru simplitate sa consideram uniformitatea densitatii de curent din plasma. Legea Maxwell-Ampere devine:

$$\frac{1}{\mu}\frac{B_\phi}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = j_z$$

Solutia este evidenta:

$$rB_\phi = \frac{1}{2}\mu r^2 j_z + C$$

Putem considera C=0 deoarece alegem ca in r=0 campul sa fie finit. Deci:

$$B_\phi = \frac{1}{2}\mu r j_z$$

Astfel, ecuatia elchilibrului magnetostatic devine o ecuatie pentru presiune:

$$-\frac{\partial p}{\partial r} = \frac{1}{\mu}\frac{B_\phi}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = \frac{1}{2}\mu j_z \frac{\partial}{\partial r}\left(\frac{1}{2}\mu r^2 j_z\right) = \frac{1}{2}\mu^2 r j_z^2$$

$$\Rightarrow p = C_1 - \frac{1}{4}\mu^2 r^2 j_z^2$$

Unde C_1 este o constanta determinabila din conditia ca plasma sa fie confinata in regiunea $r < a$, adica:

$$p(r = a) = 0 \Rightarrow p = \frac{1}{4}\left(a^2 - r^2\right)\mu^2 j_z^2$$

Cu aceasta solutie

$$p + \frac{B_\phi^2}{2\mu} = const.$$

Campul magnetic exterior *r = a*

Deoarece in vecinatatea lui r = a este vid:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = 0 \Rightarrow rB_\phi = const. \Rightarrow B_\phi = \frac{C}{r}$$

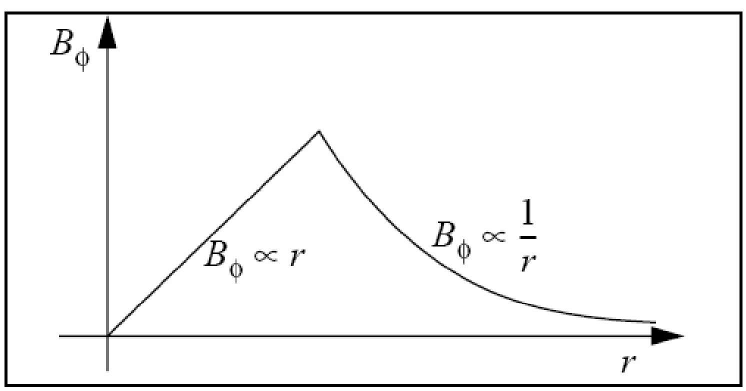
Unde constanta de integrare se determina din conditia de continuitate in r = a.

$$B_\phi = \frac{C}{a} = \frac{1}{2}\mu a j_z \Rightarrow C = \frac{1}{2}\mu a^2 j_z \Rightarrow \quad rB_\phi = \frac{1}{2}\mu a^2 j_z$$

$$\Downarrow$$

$$B_\phi = \frac{1}{2}\mu \frac{a^2}{r} j_z$$

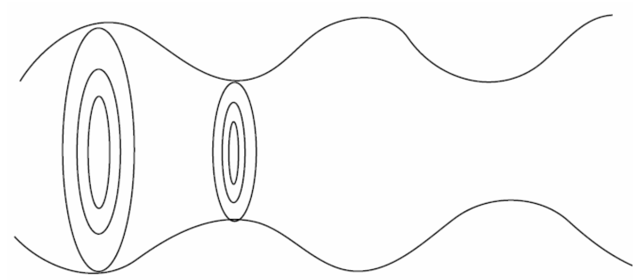
Astfel profilul radial al campului toroidal va fi:



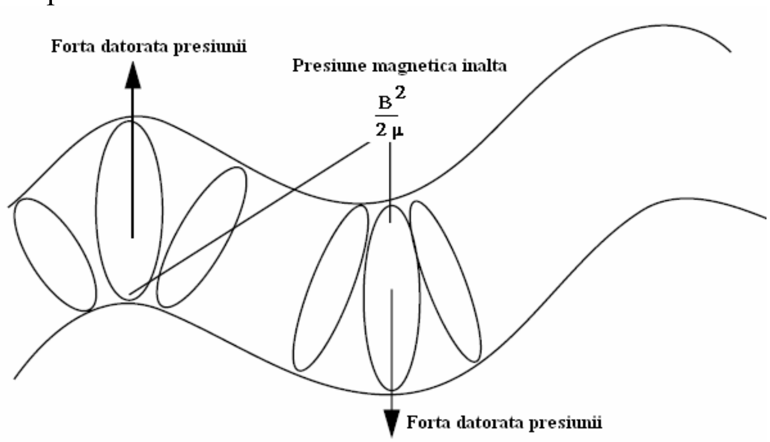
Stabilitatea pinch-ului magnetic

Pinch-ul magnetic poate fi suportul a doua tipuri de instabilitati

a) instabilitate de tip “**sausage**”



b)instabilitate de tip “**firehose**”



Θ-pinch

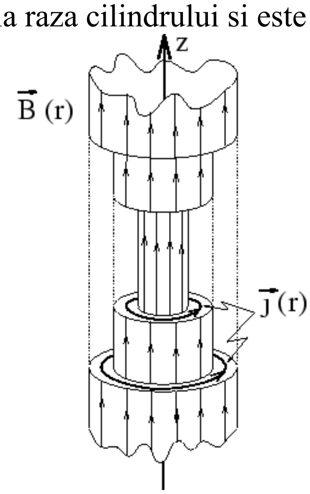
Este o structura complementara pentru o plasma cilindrica caracterizata de prezenta unui camp magnetic parallel cu axa cilindrului si care este inconjurat de o densitate de current **j** :

$$\vec{B} = B(r)\ \hat{z}$$

Unde **r** este distanta de la axul cilindrului iar $\hat{z}$ este versorul axei oz. Densitatea de curent specifica unei astfel de configuratii este de forma:

$$\vec{\mathrm{j}} = \frac{\nabla \times \vec{B}}{\mu_0} = -\frac{1}{\mu_0}\frac{dB(r)}{dr}\ \hat{\theta}.$$

Unde $\hat{\theta}$ este versorul tangentei la raza cilindrului si este perpendicular pe axa oz.



Forta Lorentz:

$$\vec{j}\times\vec{B} = -\frac{1}{2\mu_0}\ \frac{d}{dr}B^2(r)\ \hat{r}$$

Este orientate catre axa cilindrului, in timp ce intensitatea campului creste cu distanta.
Ecuatia ce descrie echilibrul este:

$$-\nabla p + \vec{j}\times\vec{B} = 0.$$

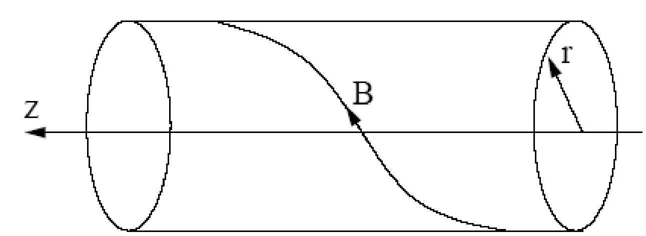
$$\nabla p = \hat{r}\ dp/dr \quad \Longrightarrow \quad \frac{d}{dr}\left(p + \frac{B^2}{2\mu_0}\right) = 0$$

Deci preiunea totala (magnetica + hidrostatica) este constanta:

$$p_{tot} = p + p_M = const.$$

"screw-pinch"

$$\mathbf{B} = B_\theta\hat{\theta}(r) + B_z\hat{z}(r),$$



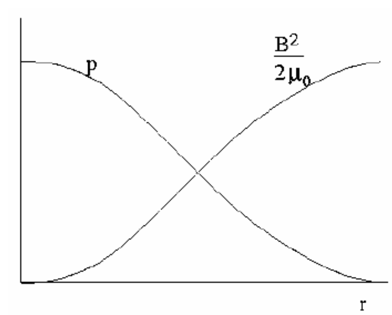
$$(\mathbf{B}\cdot\nabla)\mathbf{B} = -\frac{B_\theta^2}{r}\nabla r,$$

Iar conditia de echilibru $\vec{j}\times\vec{B} = \nabla p$ devine:

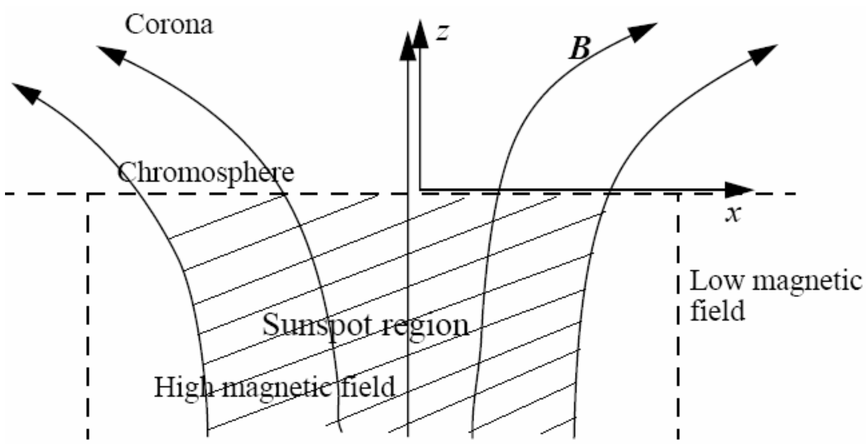
$$\left[\nabla\left(p + \frac{B^2}{2\mu_0}\right) - (\mathbf{B}\cdot\nabla)\mathbf{B}\right]\cdot\nabla r = \frac{d}{dr}\left(p + \frac{B_z^2 + B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r}$$

$$= \boxed{\frac{d}{dr}\left(p + \frac{B_z^2}{2\mu_0}\right) + \frac{B_\theta}{\mu_0 r}\frac{d(rB_\theta)}{dr} = 0}$$

Ecuatie care permite determinarea valorii componentelor B_θ, B_z necesare pentru a confina plasma la o presiune p(r) data.



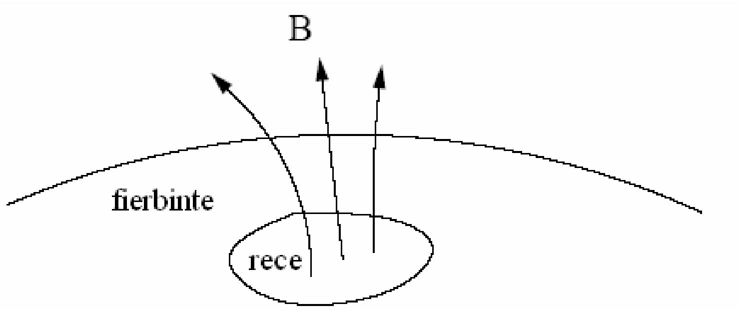
In cazul petelor solare:



$$B_\theta \simeq 0,$$

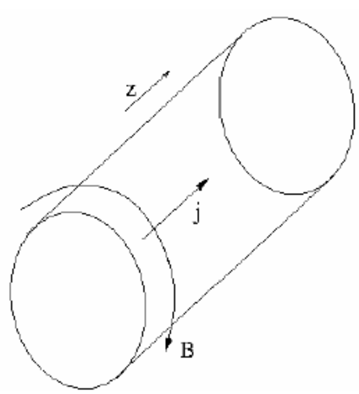
$$p + \frac{B_z^2}{2\mu_0} = \text{constant}.$$

In centrul petei solare B_z este mare iar p este mica deoarece pata solara este rece, deci mai intunecata decat vecinatatile.



$$\beta = \frac{p}{B^2/2\mu_0} = \frac{\text{thermal energy}}{\text{magnetic energy}} < 0.1$$

Z-pinch



Observam ca in aceasta configuratie **j** este orientat in lungul axei oz.

$$\mathbf{j} = \mathbf{j}_z \hat{\mathbf{e}}_z \qquad \mathbf{B} = \mathbf{B}_\theta \hat{\mathbf{e}}_\theta$$

Forta Lorentz: $$\left(\vec{j} \times \vec{B}\right)_r - \left(\nabla p\right)_r = -j_z B_\theta - \frac{\partial p}{\partial r} = 0$$

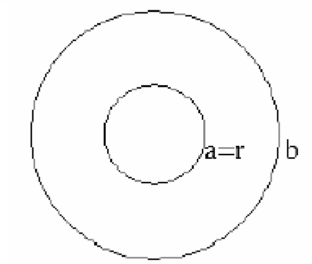
Ecuatia M-A: $$\left(\nabla \times \vec{B}\right)_z - \left(\mu_0 \vec{j}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right) - \mu_0 j_z = 0$$

$$\underbrace{\frac{B_\theta^2}{\mu_o r}}_{\text{Tensiune mg.}} + \frac{\partial}{\partial r}\underbrace{\left(\frac{B_\theta^2}{2\mu_o} + p\right)}_{\text{Pres.magnetica} \quad \text{Pres.cinetica}} = 0$$

Tensiune mg. Pres.magnetica Pres.cinetica

Primul termen se datoreaza curbarii liniilor campului magnetic. Integram ecuatia:

$$\int_a^b \frac{B_\theta^2}{\mu_o}\,\frac{dr}{r} + \left[\frac{B_\theta^2}{2\mu_o} + p(r)\right]_a^b = 0$$



Cum p(b) = 0 si a = r rezulta imediat:

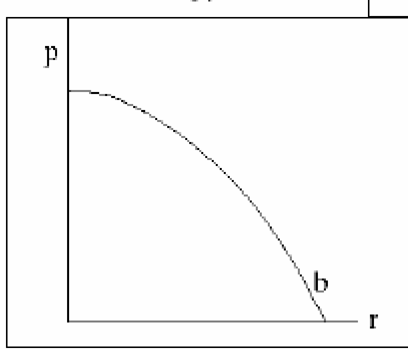
$$p(r) = \frac{B_\theta^2(b)}{2\mu_o} - \frac{B_\theta^2(r)}{2\mu_o} + \int_r^b \frac{B_\theta^2}{\mu_o}\frac{dr'}{r'}$$

Daca:

$j = \text{const.}$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_o j_z \Rightarrow B_\theta = \frac{\mu_o j_z}{2}\, r$$

$$p(r) = \frac{1}{2\mu_o}\left(\frac{\mu_o j_z}{2}\right)^2 \{b^2 - r^2 + \int_r^b 2r'dr'\} \boxed{= \frac{\mu_o j_z^2}{4}\{b^2 - r^2\}}$$



Un profil parabolic

$B_\theta(b) = \frac{\mu_o j_z b}{2}$ ➔ $$\boxed{p = \frac{B_{\theta b}^2}{2\mu_o}\frac{2}{b^2}\{b^2 - r^2\}}$$

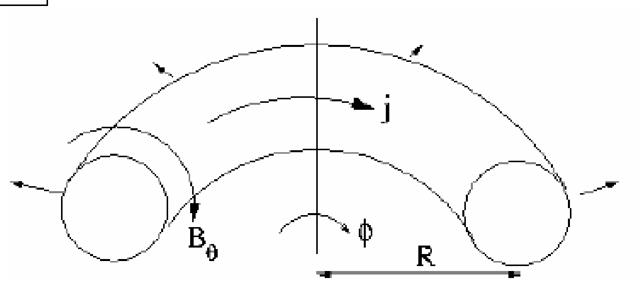
‘screw pinch’, $\theta - z$ pinch

$$j_\theta B_z - j_z B_\theta - \frac{\partial}{\partial r} = 0 \quad \frac{\partial}{\partial r}B_z = \mu_o j_\theta$$

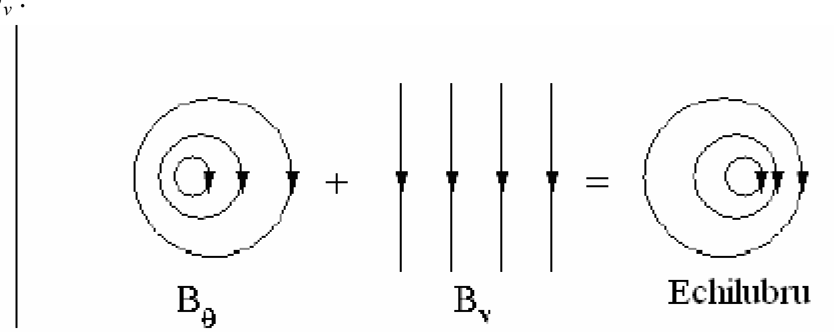
$$-\frac{B_z}{\mu_o}\frac{\partial B_z}{\partial r} - \frac{B_\theta}{\mu_o r}\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial p}{\partial r} = 0$$

$$\underbrace{\frac{B_\theta^2}{\mu_o r}}_{\substack{\text{Mag Tension}\\ \theta \text{ only}}} + \frac{\partial}{\partial r}\underbrace{\left(\frac{B^2}{2\mu_o} + p\right)}_{\text{Mag }(\theta+\text{z})+\text{Kinetic pressure}} = 0$$

Toroidal z-pinch

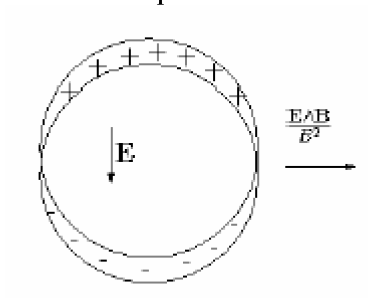


B_θ este mult mai intens in regiunile cu R mai mic => exercitarea unei presiuni magnetice, deci aparitia unei forte indreptate spre exterior. Pentru a restabili echilibrul este necesara aplicarea unui camp suplimentar exterior B_v, care “impinge’ plasma inapoi prin intermediul fortei $\vec{j}_\phi \times \vec{B}_v$.



In cazul “incovoierii” θ-pinch in tor, B_ϕ este mai intens in regiunea cu R mai mic =>existenta unei alte forte indreptate spre exterior, dar care nu mai poate fi compensata de aplicarea unui camp exterior , deoarece nu exista o componenta j_ϕ . Astfel, nu exista echilibru pentru θ-pinch toroidal simetric.

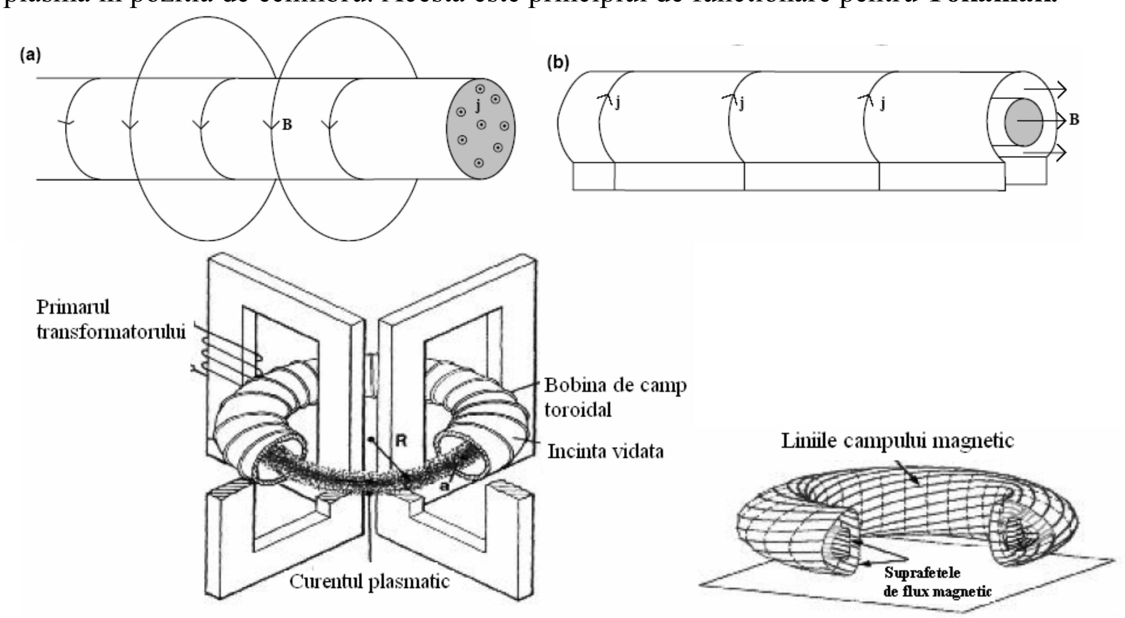
Din punctul de vedera al teoriei miscarii particulelor in EB, Θ-pinch-ul toroidal contine numai B_ϕ si asa cum am vazut anterior, drifturile de curbura raman necompensate, ceea ce va conduce la o rapida tendinta de miscare spre exterior.



Se observa o seprare de sarcina ce conduce la un drift spre exterior sau altfel spus nu exista forte de balans MHD toroidale.

Cum rezolvam aceasta problema?

Introducem o transformare de rotatie: dam un B_θ => introducem rapid un j_ϕ ceea ce din punct de vedere MHD inseamna existanta unei forte de balans $\vec{j}_\phi \times \vec{B}_v$, care va readuce plasma in pozitia de echilibru. Acesta este principiul de functionare pentru **Tokamak.**



a) Z-pinch, b) θ-pinch, c) Tokamak

Cel mai performant Tokamak este cel de la Princeton, construit in 1994 si numit **TTFR = Tokamak Test Fusion Reactor.**(Raza torului ~ 2.5m, B ~ 5T, Cu ~40MW putere de incalzire se obtine o putere nucleara de ~ 10MW, Timpul de confinare τ ~ 1s)