BAREM EXAMEN MECANICĂ.

$$\bar{v} = (2, -3t)$$

1. $\bar{v}(t) = 2\bar{i} - 3t\bar{j} \Rightarrow v_x = 2 \qquad v_y = -3t$

a) $\bar{r}(t) = (x, y) \qquad v_x = \frac{dx}{dt} \Rightarrow x(t) = \int v_x(t)\,dt = 2t + C$

$$x(0) = 0 \Rightarrow C = 0$$

$$v_y = \frac{dy}{dt} \Rightarrow y(t) = \int v_y(t)\,dt = -\frac{3t^2}{2} + C$$

$$y(0) = 0 \Rightarrow C = 0$$

$$\bar{r}(t) = \left(2t, -\frac{3t^2}{2}\right) \quad \boxed{0.5p}$$

b) $\bar{a}(t) = \frac{d\bar{v}}{dt} = -3\bar{j} = (0, -3) \quad \boxed{0.5p}$

c) $\bar{L} = \bar{r} \times \bar{p} = \cancel{k}\left(2t, -\frac{3t^2}{2}, 0\right) \times (2m, -3mt)$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2t & -\frac{3t^2}{2} & 0 \\ 2m & -3mt & 0 \end{vmatrix} = \dots \quad \boxed{0.5p}$$

d.) $\bar{v} = (2, -3t)$

$\bar{F} = m \cdot \bar{a} = m \cdot (0, -3)$

$\cos \widehat{\bar{F}, \bar{v}} = \frac{\bar{F} \cdot \bar{v}}{F \cdot v} \Rightarrow \dots \quad \boxed{0.5p}$

e.) $x(t) = 2t$

$y(t) = -\frac{3t^2}{2}$ $\Rightarrow$ $t = \frac{x}{2}$

$$\boxed{y = -\frac{3}{2}\frac{x^2}{4} = -\frac{3x^2}{8}}$$

$\boxed{0.5p}$

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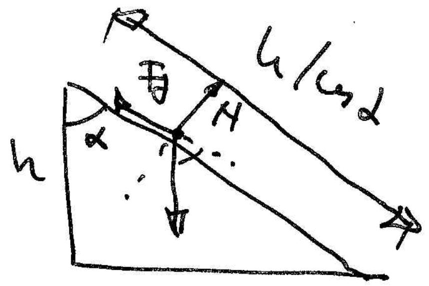
2 A1 A→B → mişcare cu frecare ⇒ variaţia energiei mecanice = L/forţe frecare.

$\Delta E_m = L_{F_f}$

$E_B - E_A = L_{F_f}$



$$\begin{cases} \frac{mv_B^2}{2} - mgh = -F_f \cdot \frac{h}{\cos\alpha} \\ F_f = \mu N \\ N = mg\sin\alpha \end{cases} \Rightarrow \underline{\underline{v_B}}$$

B→C mişcare fără frecare.

$E_B = E_C$

$$\frac{mv_B^2}{2} = \frac{mv_C^2}{2} + mgR(1-\cos\alpha) \Rightarrow \underline{\underline{v_C}}$$

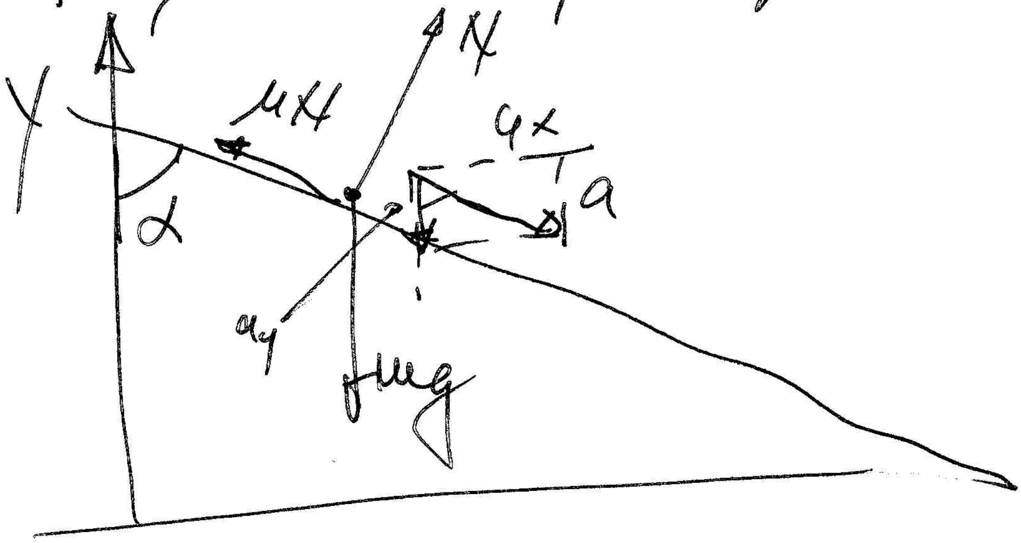
C→D mişcare fără frecare

$E_C = E_A \Rightarrow \frac{mv_C^2}{2} + mgR(1-\cos\alpha) = \frac{mv_D^2}{2} \Rightarrow$

$\Rightarrow \underline{\underline{v_D}}$ Se observă că $v_B = v_D$.

$\boxed{0.5p}$

(A2) Forțele care act. asupra corpului:



$$\left.\begin{aligned} mg\cos\alpha - \mu N &= m \cdot a \\ -mg\sin\alpha + N &= 0 \end{aligned}\right\} \Rightarrow \underline{\underline{a}}$$

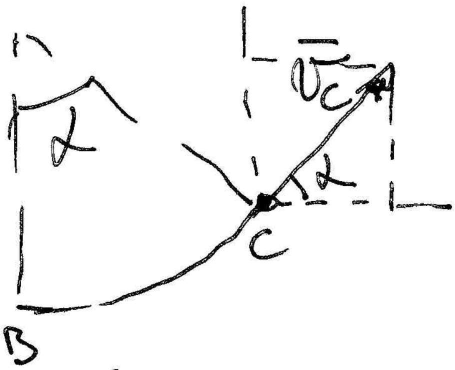
$\bar{a} = \bar{a}_x + \bar{a}_y \Rightarrow \boxed{a_y = a \cdot \cos\alpha}$.

accelerație constantă $\Rightarrow$

$y(t) = y_0 + v_{0y} \cdot t + \frac{a_y \cdot t^2}{2}$ adică

$$\boxed{y(t) = h + 0 - \frac{a\cos\alpha \cdot t^2}{2}}$$ $\boxed{0.5p}$

(A 3)



la desprindere, corpul are viteza $\overline{v_c}$ orientată sub unghiul α față de orizontală.

$$y(t) = R(1-\cancel{R}\cos\alpha) + v_0 \sin\alpha \cdot t - g\frac{t^2}{2}$$

y_0 $\qquad$ $v_{0y}\cdot t$ $\qquad$ $\frac{at^2}{2}$ $\quad a=-g$

când cade pe suprafața orizontală $\Rightarrow y=0$ și rezolvând ecuația

$R(1-\cos\alpha) + v_0 \sin\alpha t - g\frac{t^2}{2} = 0 \Rightarrow t_{CD}$ = timpul până la punctul D.

$BD = R\sin\alpha + \underbrace{v_0\cos\alpha}_{v_x} \cdot t_{CD}$ pentru că pe Ox are mișcare cu viteză constantă.

$\boxed{0.5p}$

<u>VARIANTĂ</u>: - DIN ECUAȚIA TRAIECTORIEI

(A4) $y(t) = \text{maxim} \Rightarrow$

$\frac{dy}{dt} = 0$ (viteza / oy = 0) $\Rightarrow$

$v_0 \sin\alpha - g t = 0 \Rightarrow t_{urcare}$.

$Y_{max} = Y(t_{urcare})$ $\boxed{0.5p}$

variantă

$E_c = E_{h\,max}$ $E_{h\,max} = m g h_{max} + \frac{m v_x^2}{2}$

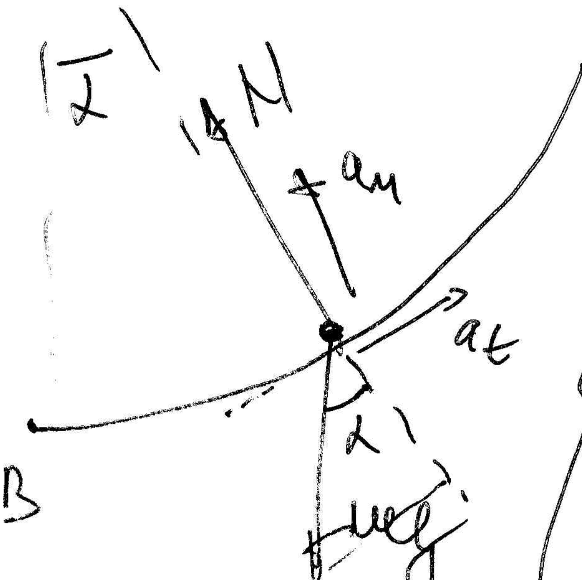
$\downarrow$

o știm

la h max se mișcă cu v_x. ($v_y = 0$).

$\parallel$

(A5,6)



$$\begin{cases} -mg\sin\alpha = m a_t \\ N - mg\cos\alpha = m a_n, \quad a_n = \frac{v^2}{R} \\ \frac{m v_B^2}{2} = mgR(1-\cos\alpha) + \frac{m v^2}{2} \end{cases}$$

$\boxed{1p}$ $\Rightarrow a_t, a_n, N$

(A7)

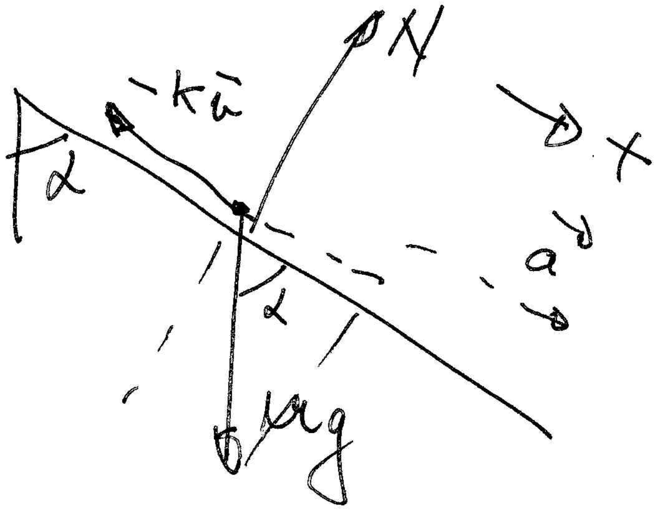
$$W_{BC} = -\Delta U_{BC} = U_B - U_C = 0 - mgh_C$$

$$= 0 - mgR(1-\cos\alpha) = -mgR(1-\cos\alpha).$$

$\boxed{0.5p}$

(B)

$$F_f = -k\bar{v}$$



$$mg\cos\alpha - kv = m\cdot a$$

$$mg\cos\alpha - kv = m\cdot\frac{dv}{dt}$$

$$\frac{m\,dv}{mg\cos\alpha - kv} = dt \Rightarrow m\int\frac{dv}{mg\cos\alpha - kv} = \int dt$$

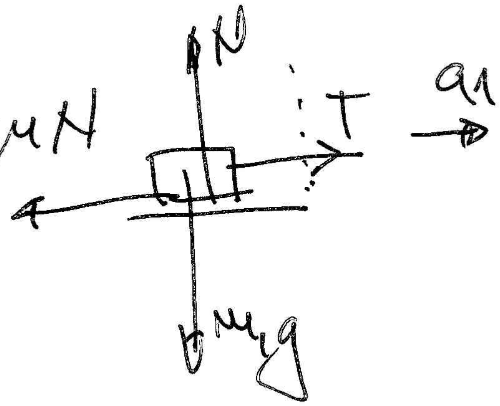
$\Rightarrow v(t)$ care nu se cere.

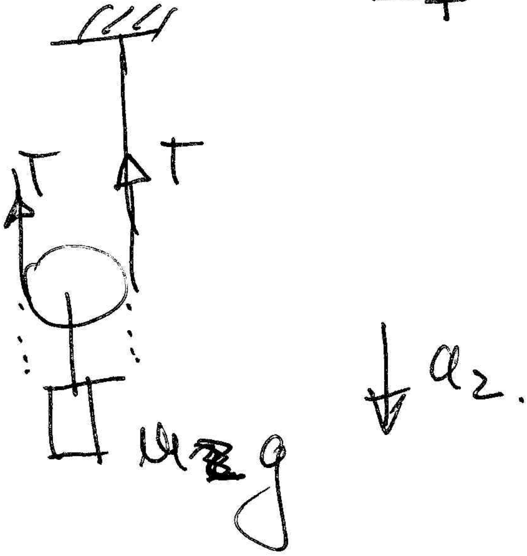
$$mg\cos\alpha - kv = m\cdot\frac{dv}{dt}\cdot\frac{ds}{ds} = m\,v\cdot\frac{dv}{ds}$$

$$\Rightarrow \int\frac{m\,v\,dv}{mg\cos\alpha - kv} = \int ds \Rightarrow v(s)$$

$\boxed{1p}$

3. $F_f = \mu N$





$$\begin{cases} T - \mu N = m_1 a_1 \\ N - m_1 g = 0 \\ m_2 g - 2T = m_2 a_2 \\ a_2 = 0.5\, a_1 \end{cases} \Rightarrow T, N, a_1, a_2$$

$\boxed{1p}$

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4. $v_x = \text{const} = c.$ $\qquad a \overset{?}{=} \dfrac{v^3}{Rc}$

$$a^2 = a_n^2 + a_t^2 = \frac{v^2}{R} + a_t^2.$$

$$a_t = \frac{dv}{dt}$$

$v = \sqrt{v_x^2 + v_y^2} \Rightarrow v^2 = c^2 + v_y^2$ / derivăm în raport cu timpul $\Rightarrow$

$$\cancel{2} v \cdot \frac{dv}{dt} = \cancel{2} v_y \cdot \frac{dv_y}{dt} \Rightarrow$$

$$\Rightarrow \frac{dv}{dt} = a_t = \frac{v_y}{v} \cdot \frac{dv_y}{dt} \quad \leftarrow a$$

dar $a = a_x + a_y \Rightarrow a = a_y$.

$a_x = 0$

$$\Rightarrow a_t = \frac{v_y}{v} \cdot a.$$

$\cancel{v^2 = c^2 +}$

$$a^2 = \frac{v^2}{R} + a_t^2$$

$$= \frac{v^2}{R} + \frac{v_y^2}{v^2} \cdot a^2$$

$$a^2 = \frac{v^2}{R} + \frac{(v^2 - c^2)}{v^2} \cdot a^2 \Rightarrow$$

$$a = \frac{v^3}{Rc} \ldots \boxed{1p}$$

P1 = 5x0.5p = 2.5p
P2A = 7x0.5p = 3.5p
P2B = 1p = 1p
P3 = 1p = 1p
P4 = 1p = 1p

TOTAL 9p + 1 oficiu

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