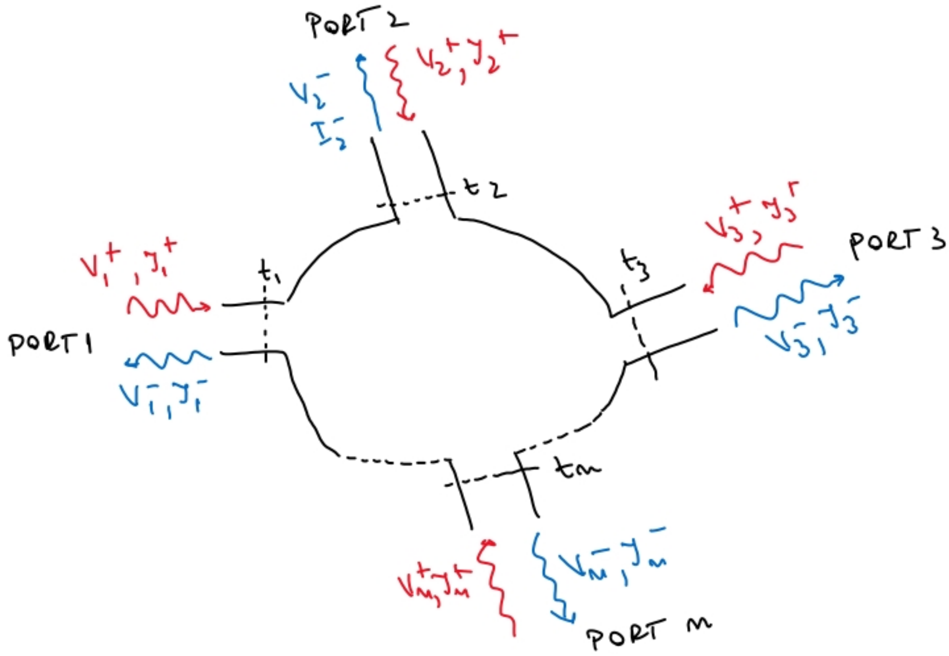
Curs 11:

Rețele de microunde:



Unda incidentă: (V_n^+, I_n^+)

Unda reflectată: (V_n^-, I_n^-)

$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ - I_n^-$$

Matricea impedanță (matricea Z)

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & & & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix}$$

$$[V] = [Z][I]$$

$$Z_{ij} = \frac{V_i}{I_j}\Big|_{I_k = 0 \text{ pt. } k \neq j}$$

↳ generăm un curent pe portul j și măsurăm tensiunea de mers în gol pe portul i

OBS: toate celelalte porturi ($k \neq j$) sunt configurate pt. mers în gol („open-circuited").

Z_{ii} - impedanța de intrare a portului i atunci când toate celelalte porturi sunt în gol.

Matricea admitanță (matricea Y)

$$[Y] = [Z]^{-1}$$

$$[I] = [Y][V]$$

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \vdots & & & \vdots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix}$$

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \text{ pt. } k \neq j}$$

↳ aplicăm o tensiune pe portul j și măsurăm curentul de scurtcircuit pe portul i.

OBS: toate celelalte porturi ($k \neq j$) sunt scurtcircuitate!

În general Z_{ij}, Y_{ij} sunt complexe.

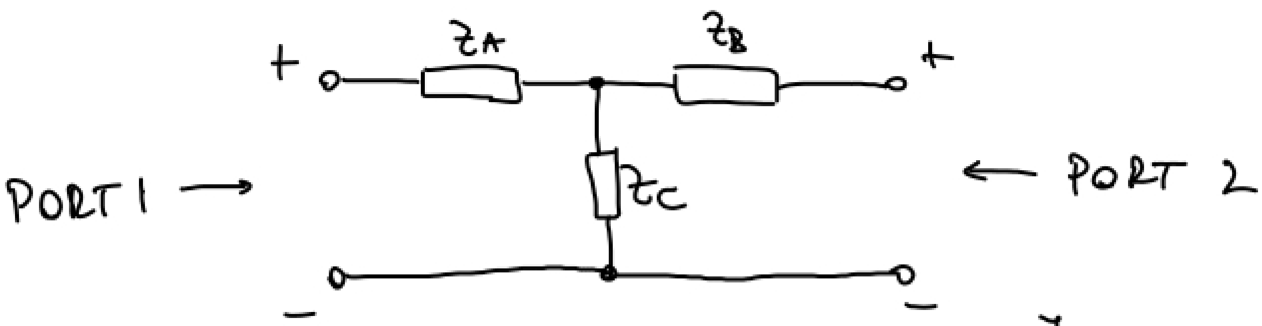
Pentru o rețea reciprocă:

$$Z_{ij} = Z_{ji} \; ; \; Y_{ij} = Y_{ji} .$$

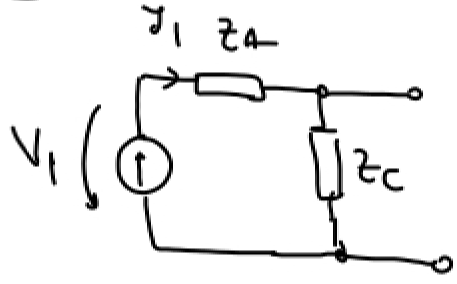
Dacă rețeaua este fără pierderi

Z_{ij}, Y_{ij} → imaginare.

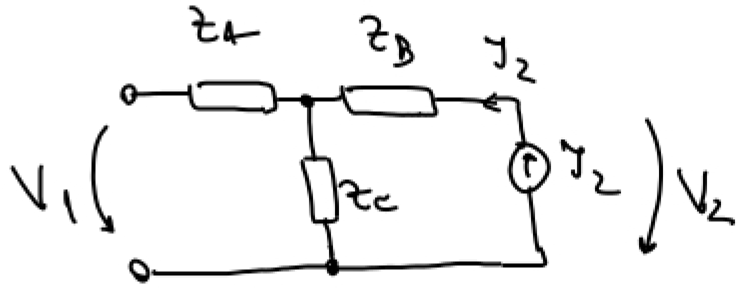
(P1.) Determinați elementele z_{ij} ale matricii Z pentru rețeaua de mai jos.



$$z_{11} = \frac{V_1}{J_1}\bigg|_{J_2=0} = Z_A + Z_C$$



$$z_{12} = \frac{V_1}{J_2}\bigg|_{J_1=0}$$



$$z_{12} = \frac{V_2}{J_2} \cdot \frac{Z_C}{Z_B+Z_C} =$$

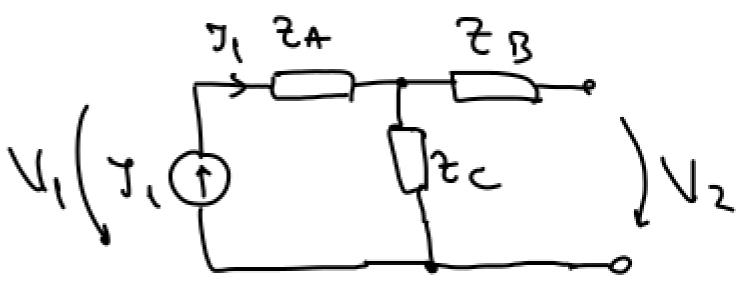
$$= \cancel{(Z_B+Z_C)} \cdot \frac{Z_C}{\cancel{Z_B+Z_C}} = Z_C$$

$$V_2 = J_2\,(Z_B+Z_C)$$

$$V_1 = J_2 \cdot Z_C$$

$$\frac{V_1}{V_2} = \frac{\cancel{J_2}\, Z_C}{(Z_B+Z_C)\cancel{J_2}} \Rightarrow V_1 = V_2 \cdot \frac{Z_C}{Z_B+Z_C}$$

$$z_{21} = \frac{V_2}{J_1}\bigg|_{J_2=0}$$



$$V_2 = J_1 z_C$$

$$V_1 = J_1(z_A + z_C)$$

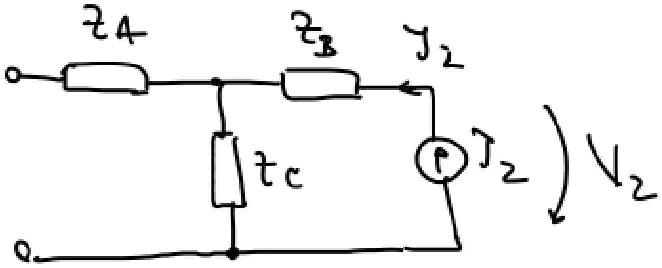
$$\frac{V_2}{V_1} = \frac{\cancel{J_1} z_C}{\cancel{J_1}(z_A+z_C)}$$

$$V_2 = V_1 \cdot \frac{z_C}{z_A+z_C}$$

$$z_{21} = \frac{V_1}{J_1} \cdot \frac{z_C}{z_A+z_C} = \cancel{(z_A+z_C)}\frac{z_C}{\cancel{z_A+z_C}}$$

$$z_{21} = z_C = z_{12}$$

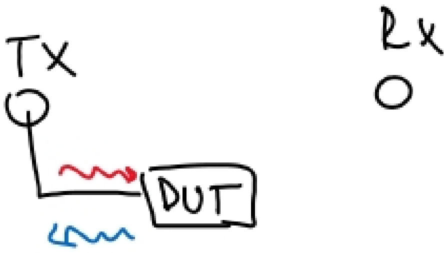
$$z_{22} = \left.\frac{V_2}{J_2}\right|_{J_1=0}$$



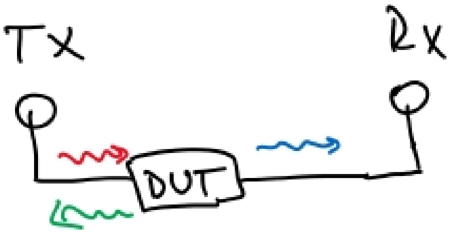
$$z_{22} = z_B + z_C = z_{11}$$

În laborator → Vector Network Analyzer

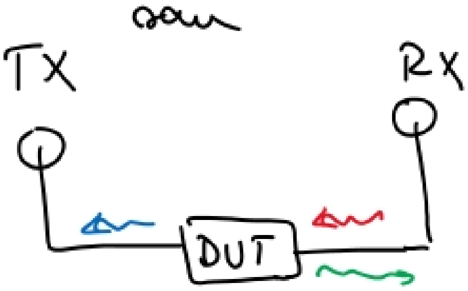
Reflexie



Transmisie



sau



Matricea S ("scattering parameters").

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \dots & S_{1N} \\ S_{21} & S_{22} & \dots & S_{2N} \\ \vdots & & & \vdots \\ S_{N1} & S_{N2} & \dots & S_{NN} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{pmatrix}$$

$$[V^-] = [S]\,[V^+]$$

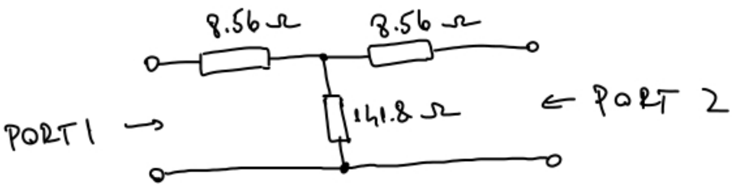
$$S_{ij} = \frac{V_i^-}{V_j^+}\Bigg|_{V_k^+ = 0 \text{ pt. } k \neq j}$$

↳ generăm o undă incidentă pe portul j și măsurăm unda reflectată (sau transmisă) din portul i.

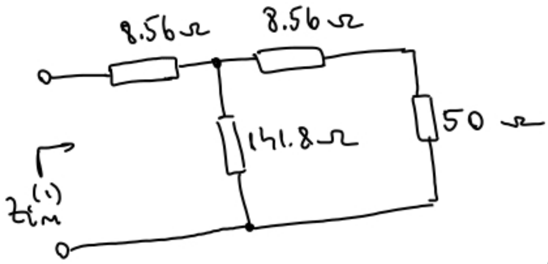
OBS: - toate celelalte porturi sunt terminate într-o impedanță adaptată.
- nu avem unde incidente pe porturile $k \neq j$

$S_{ii} = \Gamma$.

P2. Găsiți elementele matricii S pentru atenuatorul de 3 dB din figura de mai jos. ($Z_0 = 50\ \Omega$)



$$S_{11} = \frac{V_1^-}{V_1^+}\Bigg|_{V_2^+ = 0} = \Gamma^{(1)}\Big|_{V_2^+ = 0} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}\Bigg|_{Z_0 \text{ pe port } 2}$$

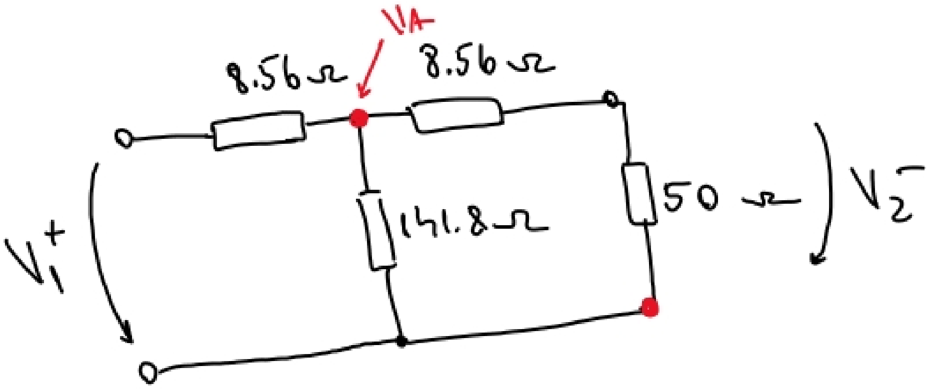


$Z_{in}^{(1)} = 50.02\ \Omega \Rightarrow \Gamma^{(1)} \simeq 0.$

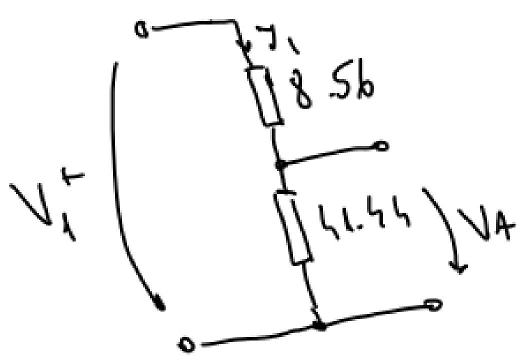
$S_{11} = 0$

$S_{22} = 0$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$



$$V_2^- = \frac{50}{58.56} \cdot V_A$$



$$V_1^+ = I_1 (8.56 + 41.44)$$

$$V_A = I_1 \cdot 41.44$$

$$\frac{V_A}{V_1^+} = \frac{41.44}{50}$$

$$V_A = V_1^+ \cdot \frac{41.44}{50}$$

$$V_2^- = \frac{\cancel{50}}{58.56} \cdot \frac{41.44}{\cancel{50}} \cdot V_1^+$$

$$S_{21} = \frac{V_2^-}{V_1^+} = \frac{41.44}{58.56} = 0.707.$$

$$S_{12} = 0.707$$

Rețea reciprocă $\Rightarrow$ matricea S este simetrică

$$[S] = [S]^t$$

Rețea fără pierderi:

dacă $i = j$

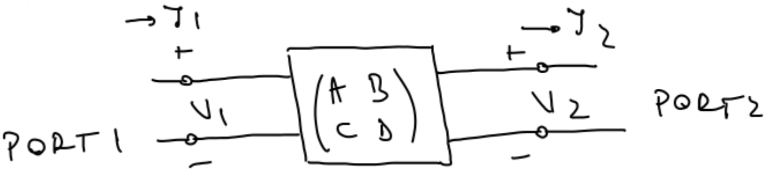
$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1$$

dacă $i \neq j$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0$$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}$$

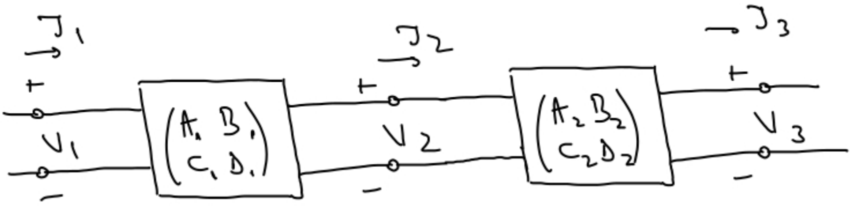
Matricea ABCD de transmisie:



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

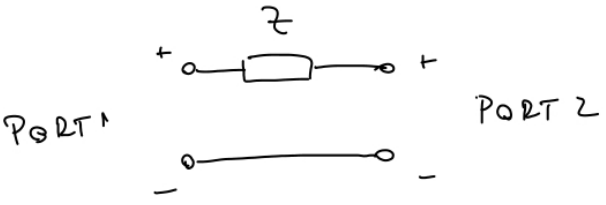
$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

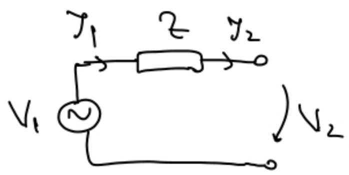


$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} V_3 \\ I_3 \end{pmatrix}$$

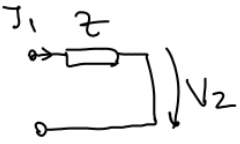
(P3) Determinați elementele A B C D pentru schema de mai jos.



$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} = Z$$



$$B = \frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{V_1}{\frac{V_1}{Z}} = \frac{\cancel{V_1}}{\cancel{V_1}} \cdot Z$$



$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} = 0$$

$$D = \frac{I_1}{I_2}\bigg|_{V_2=0} = 1.$$