Curs 12

Ghiduri de undă.

Ghiduri de undă cu 2 conductori (cablu bifilar, coaxial etc.) → TEM, TE, TM

Ghiduri de undă cu 1 conductor (ghid dreptunghiular, circular) → TE, TM

Soluții generale pt. modurile TEM, TE și TM:

$$\bar{E}(x,y,z) = [\bar{e}(x,y) + \hat{z}\, e_z(x,y)]\, e^{-j\beta z}$$

$$\bar{H}(x,y,z) = [\bar{h}(x,y) + \hat{z}\, h_z(x,y)]\, e^{-j\beta z}$$

$\bar{e}(x,y)$, $\bar{h}(x,y)$ - componentele transversale ale câmpurilor electrice și magnetice

e_z, h_z - componentele longitudinale ale câmpurilor electrice și magnetice

Ec. Maxwell:

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$H_x = \frac{j}{k_c^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x}\right)$$

$$H_y = -\frac{j}{k_c^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y}\right)$$

$$E_x = -\frac{j}{k_c^2}\left(\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{j}{k_c^2}\left(-\beta \frac{\partial E_z}{\partial y} + \omega\mu \frac{\partial H_z}{\partial x}\right)$$

$k_c^2 = k^2 - \beta^2$ - „cutoff wavenumber"; $k_c = \sqrt{k^2-\beta^2}$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

Undele (modurile) TEM (Transverse ElectroMagnetic)

TEM: $E_z = 0$
$H_z = 0$

$$\beta^2 E_y = \omega^2 \mu \varepsilon E_y$$

$$\beta = \omega\sqrt{\mu\varepsilon} = k$$

$$k_c = \sqrt{k^2 - \beta^2} = 0 \Rightarrow f_{min} = 0$$

Ec. Helmholtz pt. E_x

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) E_x = 0$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\beta^2 E_x = -k^2 E_x$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) E_x = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) E_y = 0$$

$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\begin{cases} \nabla_t^2 \bar{e}(x,y) = 0 \\ \nabla_t^2 h(x,y) = 0 \end{cases}$$

Dacă avem un potențial scalar

$$\bar{e}(x,y) = -\nabla_t \phi(x,y)$$

$$\nabla_t = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}$$

$$\nabla_t \times \bar{e} = -j\omega\mu h_z \hat{z} = 0$$

$$\nabla \cdot \vec{D} = \varepsilon \cdot \nabla_t \bar{e} \Rightarrow$$

$$\Rightarrow \nabla_t^2 \phi(x,y) = 0$$

Tensiunea dintre cei doi conductori:

$$V_{12} = \phi_1 - \phi_2 \int_1^2 \bar{E}\, d\bar{l}$$

$$I = \oint_C \bar{H}\, d\bar{l}$$

C – conturul secțiunii transversale

$$Z_{TEM} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

Modurile TE: „Transverse Electric"

$$E_z = 0 , \quad H_z \neq 0$$

$$H_x = -\frac{j\beta}{k_c^2} \cdot \frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{j\beta}{k_c^2} \cdot \frac{\partial H_z}{\partial y}$$

$$E_x = -\frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x}$$

$$k_c \neq 0 \quad ; \quad \beta = \sqrt{k^2 - k_c^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) H_z = 0$$

$$H_z(x,y,z) = h_z(x,y) e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0$$

$$k_c = \sqrt{k^2 - \beta^2}$$

$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$$

Modurile TM („Transverse magnetic")

$H_z = 0; \; E_z \neq 0$

$$H_x = \frac{j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial y}$$

$$H_y = -\frac{j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial x}$$

$$E_x = -\frac{j\beta}{k_c^2} \frac{\partial E_z}{\partial x}$$

$$E_y = -\frac{j\beta}{k_c^2} \frac{E_z}{\partial y}$$

$k_c \neq 0 \; ; \; \beta = \sqrt{k^2 - k_c^2}$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) E_z = 0$$

$$E_z(x,y,z) = e_z(x,y) e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z = 0$$

$$Z_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}$$

Pierderi:

$\alpha = \alpha_c + \alpha_d$

α_c - pierderi prin conducție

α_d - pierderi prin dielectric

Pt. unde TE sau TM:

$$\alpha_d = \frac{k^2 \, tg\delta}{2\beta} \quad [Np/m]$$

Pt. unde TEM:

$$\alpha_d = \frac{k \, tg\delta}{2} \quad [Np/m]$$

P1. Ghid de undă dreptunghiular din Cu având

$a = 1.07$ cm

$b = 0.43$ cm

umplut cu teflon ($\varepsilon_r = 2.08$; $tg\delta = 0.0004$).

a) Determinați f_c pentru primele 5 moduri de propagare

b) Dacă $f = 15$ GHz determinați α_d și α_c

$$k_c = \frac{2\pi}{\lambda_c} \Rightarrow \lambda_c = \frac{2\pi}{k_c}$$

$$v_p = \frac{\lambda_c}{T_c} = \lambda_c \cdot f_c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{c}{2\pi\sqrt{\varepsilon_r}} \cdot k_c$$

$$f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$\cancel{TE_{00}}$, $\cancel{TM_{00}}$, $\cancel{TM_{01}}$, $\cancel{TM_{10}}$

Moduri TE:

$$TE_{10}: f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}}$$

Mod	m	n	f (GHz)
TE_{10}	1	0	9.71
TE_{20}	2	0	19.43
TE_{01}	0	1	24.17
TE_{11}, TM_{11}	1	1	26.05
TE_{21}, TM_{21}	2	1	31.01

b) $\alpha_d = \frac{k^2 \operatorname{tg}\delta}{2\beta}$

$\operatorname{tg}\delta = 0.0004$

$k = \omega\sqrt{\mu\varepsilon} = 2\pi f \cdot \sqrt{\mu_0 \varepsilon_0 \varepsilon_r} =$

$= \frac{2\pi}{c} \cdot f \cdot \sqrt{\varepsilon_r} \cdot)$

$\Rightarrow k = 453.17 m^{-1}$

$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

TE_{10} - singurul mod de propagare la $f = 15 GHz$

$\Downarrow$

$\beta = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} =$

$= \sqrt{453.17^2 - 293.46^2} =$

$= 345.32 \ m^{-1}$

$\alpha_d = \frac{453.17^2 \times 0.0004}{2 \times 345.32} =$

$= 0.119 \ Np/m = 1.03 dB/m$

$\alpha_c = \frac{R_s}{a^3 b \beta k \eta} (2b\pi^2 + a^3 k^2) \ [Np/m]$

$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 0.032 \ \Omega$

pt. Cu $\sigma = 5.8 \times 10^7 \ S/m$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} =$

$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{2.08}} =$

$= \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} \cdot \frac{1}{\sqrt{2.08}} =$

$= 1.19 \times 316.23 \cdot \frac{1}{\sqrt{2.08}} =$

$= 376.31 \times 0.693 = 260.92 \ \Omega$

$\alpha_c = 0.05 \ Np/m = 0.43 \ dB/m$

$\alpha = \alpha_c + \alpha_d = 1.03 + 1.43 = 2.46 \ dB/m$

P.2 stripline $Z_0 = 50\,\Omega$

$b = 0.32$ cm

$\varepsilon_r = 2.2$

$tg\,\delta = 0.001$

$f = 10$ GHz

$t = 0.01$ mm

$\sqrt{\varepsilon_r}\, Z_0 = 74.16 < 120\,\Omega \Rightarrow$

$\Rightarrow \frac{W}{b} = x$

$x = \frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0}$

$W = xb = \frac{b \cdot 30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0.441 \cdot b$

$= \frac{30\pi \cdot 0.32 \times 10^{-2}}{74.16} - 0.441 b$

$= 0.00406$ m $- 0.00141$ m $=$

$= 2.65$ mm

$\alpha_d = \alpha_d(TEM) = \frac{k\, tg\,\delta}{2}$

$k = \omega\sqrt{\mu\varepsilon} = 2\pi f \cdot \sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_r}$

$= \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_r} =$

$= \frac{2\pi \times 10^{10}}{3 \times 10^8} \times 1.48 = 309.81\ m^{-1}$

$\alpha_d = \frac{309.81 \times 0.001}{2} = 0.155$ Np/m