Curs 2 microunde:

Unde plane într-un mediu cu pierderi

momente de dipol → polarizare suplimentară P_e

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}_e$$

Într-un mediu liniar $P_e = \varepsilon_0 \chi_e \cdot \vec{E}$

χ_e - susceptibilitatea electrică

$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \underbrace{\varepsilon_0 (1+\chi_e)}_{\varepsilon} \vec{E} = \varepsilon \cdot \vec{E}.$$

ε poate fi scris în formă complexă:

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0 (1+\chi_e)$$

ε'' - pierderi prin căldură din mediu datorită atenuării momentelor de dipol vibrante

mediu cu pierderi → ε complex

mediu fără pierderi → ε real

Pierderi printr-un conductor

→ conductor având conductivitatea σ

$\vec{J} = \sigma \cdot \vec{E}$ - legea lui Ohm.

$$\nabla \times \vec{H} = j\omega\vec{D} + \vec{J} = j\omega\varepsilon\vec{E} + \sigma\vec{E} = j\omega(\varepsilon' - j\varepsilon'')\vec{E} + \sigma\vec{E} =$$

$$= (j\omega\varepsilon' - j^2\omega\varepsilon'')\vec{E} + \sigma\vec{E} = j\omega\varepsilon'\vec{E} + \omega\varepsilon''\vec{E} + \sigma\vec{E}$$

$$\nabla \times \vec{H} = j\omega\varepsilon'\vec{E} + (\omega\varepsilon'' + \sigma)\vec{E}$$

$$\nabla \times \vec{H} = j\omega\underbrace{\left(\varepsilon' - j\varepsilon'' - j\frac{\sigma}{\omega}\right)}\vec{E}$$

$$\nabla \times \vec{H} = j\omega\left[\varepsilon' - j\underbrace{\left(\varepsilon'' + \frac{\sigma}{\omega}\right)}_{\text{pierderi sub formă de căldură}}\right]\vec{E}$$

$\omega\varepsilon''$ - pierderi datorită atenuării prin dielectric

σ - pierderi prin conductivitate

$\omega\varepsilon'' + \sigma$ → conductivitatea efectivă totală

Definim tangenta de pierderi

$$\boxed{tg\,\delta = \frac{\omega\varepsilon'' + \sigma}{\omega\varepsilon'}}$$

În datasheet: $\underline{\varepsilon_r + tg\,\delta}$ la o frecvență dată

Considerăm un mediu cu pierderi, cu conductivitate σ. (fără surse)

$$[\sigma] = S/m$$

$$\begin{cases} \nabla \times \vec{E} = -j\omega\mu\vec{H} \\ \nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \vec{J} \end{cases}$$

$$\begin{cases} \nabla \times \vec{E} = -j\omega\mu\vec{H} \\ \nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \sigma\vec{E} \quad (*) \end{cases}$$

Ecuațiile Helmholtz:

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times (-j\omega\mu\vec{H})$$

$$\nabla \times (\nabla \times \vec{E}) = -j\omega\mu(\nabla \times \vec{H}) \overset{(*)}{\Rightarrow} \nabla \times (\nabla \times \vec{E}) = -j\omega\mu(j\omega\varepsilon\vec{E} + \sigma\vec{E})$$

$$\nabla \times (\nabla \times \vec{E}) = -j^2\omega^2\mu\varepsilon\vec{E} - j\omega\mu\vec{E}$$

$$\left[\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A} \right] \Bigg| \Rightarrow$$

$$\Rightarrow \nabla(\cancelto{0}{\nabla\vec{E}}) - \nabla^2\vec{E} = -j^2\omega^2\mu\varepsilon\vec{E} - j\omega\mu\sigma\vec{E}$$

$$-\nabla^2 E = -j^2\omega^2\mu\varepsilon\vec{E} - j\omega\mu\sigma\vec{E}$$

$$\nabla^2\vec{E} = j^2\omega^2\mu\varepsilon\vec{E}\left(1 + \frac{\cancel{j}\cancel{\omega}\cancel{\mu}\sigma}{j^{\cancel{2}}\omega^{\cancel{2}}\cancel{\mu}\varepsilon}\right)$$

$$\nabla^2\vec{E} = j^2\omega^2\mu\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)\vec{E}$$

$$\nabla^2\vec{E} = -\omega^2\mu\varepsilon\left(1 + \frac{j\sigma}{jj\omega\varepsilon}\right)\vec{E}$$

$$\boxed{\nabla^2\vec{E} + \underbrace{\boxed{\omega^2\mu\varepsilon\left(1 - j\cdot\frac{\sigma}{\omega\varepsilon}\right)}}_{-\gamma^2}\vec{E} = 0}$$

ecuația undei într-un mediu cu pierderi

Mediu fără pierderi:

$$\nabla^2\vec{E} + k^2\vec{E} = 0$$

$k^2 = \omega^2\mu\varepsilon$ - constanta de propagare

Definim constanta de propagare complexă

$$\gamma^2 = j^2\omega^2\mu\varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]$$

$$\boxed{\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

α - constanta de atenuare

β - constanta de fază

Presupunem $\vec{E} = \begin{pmatrix} E_x(z) \\ 0 \\ 0 \end{pmatrix}$

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

ecuația caracteristică

$$y'' - \gamma^2 y = 0$$

ecuația auxiliară

$$r^2 - \gamma^2 = 0$$

$$r^2 = \gamma^2$$

$$r = \pm \gamma$$

$$y = c_1 e^{r_1 z} + c_2 e^{r_2 z}$$

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}$$

$e^{\pm \gamma z}$ – factorul de propagare

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$

$$\mathcal{E}_x = E_x(z,t) = E_x(z)\cos(\omega t)$$

$$\mathcal{E}_x = E^+ e^{(-\gamma z + j\omega t)} + E^- e^{(\gamma z + j\omega t)} =$$

$$= E^+ e^{-\alpha z - j\beta z + j\omega t} + E^- e^{\alpha z + j\beta z + j\omega t}$$

$$\mathcal{E}_x = \underbrace{E^+ e^{-\alpha z} \cdot e^{j(\omega t - \beta z)}}_{\text{unda propagată pe direcția } +Oz \text{ care se atenuează}} + E^- e^{\alpha z} e^{j(\omega t + \beta z)}$$

Viteza de propagare:

$$v_p = \frac{dz}{dt}$$

$$\omega t - \beta z = \text{const.}$$

$$-\beta z = \text{const.} - \omega t$$

$$z = \frac{\omega t - \text{const.}}{\beta}$$

$$v_p = \frac{d}{dt}\left(\frac{\omega t - \text{const.}}{\beta}\right)$$

$$\boxed{v_p = \frac{\omega}{\beta}}$$

Lungimea de undă

$$(\omega t - \beta z) - [\omega t - \beta(z + \lambda)] = 2\pi$$

$$\omega t - \beta z - \omega t + \beta z + \beta\lambda = 2\pi$$

$$\beta\lambda = 2\pi$$

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

Partea magnetică:

$$\vec{B} = \mu_0 (\vec{H} + \vec{P}_m)$$

$\vec{P}_m$ - polarizarea magnetică (magnetizarea)

$$\vec{P}_m = \chi_m \cdot \vec{H}$$

χ_m - susceptibilitatea magnetică complexă

$$\vec{B} = \mu_0 (\vec{H} + \chi_m \vec{H})$$

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m) = \mu' - j\mu''$$

μ'' - pierderi datorate forțelor de atenuare

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) = -j\omega\mu H_y$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y$$

$$\left.\begin{aligned} H_y &= -\frac{1}{j\omega\mu}\frac{\partial E_x}{\partial z} \\ E_x &= E^+ e^{-\gamma z} + E^- e^{\gamma z} \end{aligned}\right\} \Rightarrow \quad \left[-\frac{1}{j} = -\frac{j}{jj} = -\frac{j}{j^2} = j \right]$$

$$\Rightarrow \quad H_y = \frac{j}{\omega\mu}\left(E^+(-\gamma)e^{-\gamma z} + \gamma E^- e^{\gamma z}\right)$$

$$H_y = \underbrace{-\frac{j\gamma}{\omega\mu}}_{\frac{1}{\eta}}\left(E^+ e^{-\gamma z} - E^- e^{\gamma z}\right)$$

$$\frac{1}{\eta} = -\frac{j\gamma}{\omega\mu} \Rightarrow \boxed{\eta = -\frac{\omega\mu}{j\gamma}}$$ impedanța complexă a mediului

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{\frac{\sigma}{j\omega\varepsilon}} =$$

$$= (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta_S = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

↓

skin depth
(adâncime de pătrundere)