Curs 3 microunde:

Analiza liniilor de transmisie:

•) Dacă $\lambda \gg D \rightarrow$ circuite cu parametri concentrați ("lumped circuit elements")
(D - dimensiunea componentelor)

$\rightarrow$ teoria standard $\rightarrow$ aproximație a teoriei electromagnetice descrisă de ecuațiile lui Maxwell

$\boxed{U, I = f(t)}$ $\rightarrow$ variații de fază ale U, I nesemnificative pe distanța D

•) Dacă $\lambda \ll D$ - optică geometrică

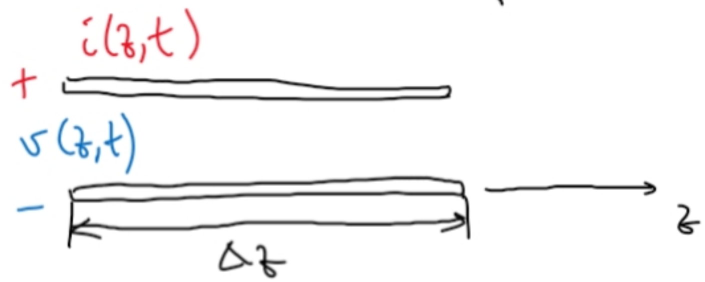
•) Dacă $\boxed{\lambda \gtrsim D}$ - circuite cu parametri distribuiți ("distributed circuit elements")

$\boxed{U, I = f(t, z)}$ - variații semnificative ale fazelor U, I pe dimensiunea componentelor de circuit.

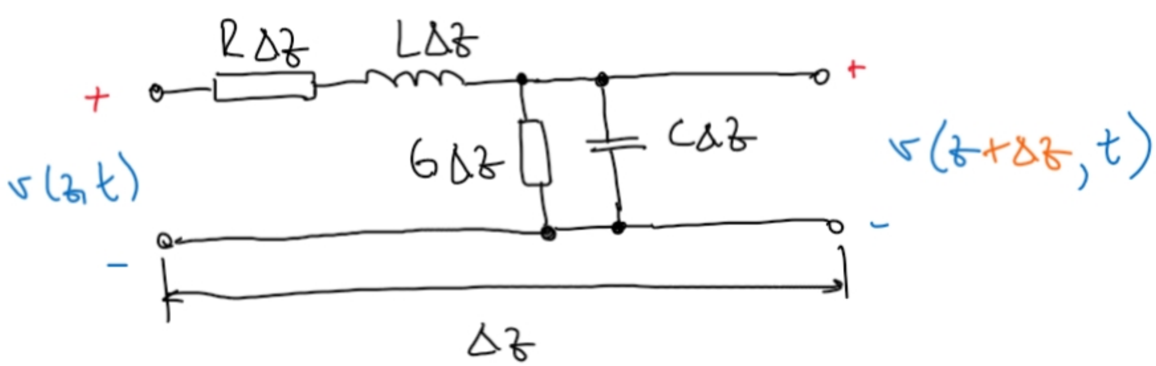
z - lungime

2 conductori → moduri TEM

Considerăm un element infinitezimal de linie → Δz.



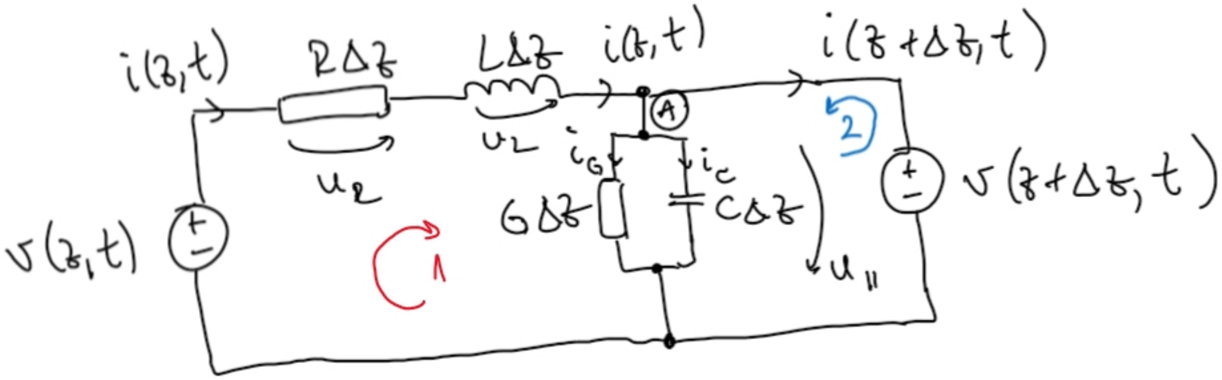
schema echivalentă a elementului infinitezimal de linie



pierderi:
- $\boxed{R\ [\Omega/m]}$ → conductivitate finită a conductorilor
- $L\ [H/m]$ → inductanța conductorilor
- $\boxed{G\ [S/m]}$ → pierderi prin dielectricul dintre conductori
- $C\ [F/m]$ → proximitate între conductori

$S = \frac{1}{\Omega} = \text{mho}$

Schema echivalentă 1:



ochiul 1:
$$v(z,t) - i(z,t)\cdot R\Delta z - L\Delta z\,\frac{\partial i(z,t)}{\partial t} - \underbrace{v(z+\Delta z,t)}_{u_{11}} = 0 \quad \Big/ \cdot \frac{1}{\Delta z}$$

⌈ pentru inductori:

$$v = L\cdot\frac{\partial i}{\partial t}$$ ⌋

ochiul 2: $v(z+\Delta z,t) = u_{11}$

Aplicăm Kirchhoff în nodul (A)

$$i(z,t) = i(z+\Delta z,t) + (i_G + i_C)$$

$$i_G \cdot \frac{1}{G\Delta z} = v(z+\Delta z,t)$$

⌈ $i = C\cdot\frac{\partial v}{\partial t}$ ⌋

$$i_C = C\Delta z\cdot\frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$i_G = G\Delta z\cdot v(z+\Delta z,t)$$

$$i(z,t) - G\Delta z\, v(z+\Delta z,t) - C\Delta z\cdot\frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \quad \Big/ \cdot \frac{1}{\Delta z}$$

$$\begin{cases} \dfrac{v(z,t)}{\Delta z} - R \cdot i(z,t) - L \cdot \dfrac{\partial i(z,t)}{\partial t} - \dfrac{v(z+\Delta z,t)}{\Delta z} = 0 \\[2ex] \dfrac{i(z,t)}{\Delta z} - G \cdot v(z+\Delta z,t) - C \cdot \dfrac{\partial v(z+\Delta z,t)}{\partial t} - \dfrac{i(z+\Delta z,t)}{\Delta z} = 0 \end{cases}$$

OBS: $\Delta z \to 0$

$$\frac{v(z,t) - v(z+\Delta z,t)}{\Delta z} - R\, i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} = 0$$

$$\underbrace{\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}}_{\frac{\partial v(z,t)}{\partial z}} = - R \cdot i(z,t) - L \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial v(z,t)}{\partial z} = - R\, i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = - G\, v(z+\underbrace{\Delta z}_{0},t) - C \cdot \frac{\partial v(z+\overset{0}{\Delta z},t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = - G\, v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t}$$

$$\begin{cases} \dfrac{\partial v(z,t)}{\partial z} = -R\,i(z,t) - L\cdot\dfrac{\partial i(z,t)}{\partial t} \\ \dfrac{\partial i(z,t)}{\partial z} = -G\cdot v(z,t) - C\cdot\dfrac{\partial v(z,t)}{\partial t} \end{cases}$$

ecuațiile telegrafiștilor.

"ecuațiile liniilor de transmisie

i, v - sinusoidale

$v(z,t) = V_0 e^{j\omega t} = V(z)$

$i(z,t) = I_0\cdot e^{j\omega t} = I(z)$

$I(z)$

$$\begin{cases} \dfrac{dV(z)}{dz} = -R\cdot I(z) - L\cdot I_0\cdot j\omega\cdot e^{j\omega t} \\ \dfrac{dI(z)}{dz} = -G\cdot V(z) - C\cdot j\omega\cdot V_0\cdot e^{j\omega t} \end{cases}$$

$V(z)$

$$\begin{cases} \dfrac{dV(z)}{dz} = -(R+j\omega L)\cdot I(z) \\ \dfrac{dI(z)}{dz} = -(G+j\omega C)\cdot V(z) \end{cases}$$

Propagarea undelor prin linia de transmisie:

$$\begin{cases} \dfrac{dV(z)}{dz} = -(R + j\omega L) \cdot I(z) \\ V(z) = -\dfrac{1}{G + j\omega C} \cdot \dfrac{dI(z)}{dz} \end{cases}$$

$$\frac{d}{dz}\left(-\frac{1}{G+j\omega C} \cdot \frac{dI(z)}{dz}\right) = -(R+j\omega L) I(z)$$

$$-\frac{1}{G+j\omega C} \cdot \frac{d^2 I(z)}{dz^2} = -(R+j\omega L) \cdot I(z)$$

$$\frac{d^2 I(z)}{dz^2} = \underbrace{(G+j\omega C)(R+j\omega L)}_{\gamma^2} I(z)$$

$$\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0$$

γ – constanta de propagare complexă

$$\gamma = \sqrt{(G+j\omega C)(R+j\omega L)}$$

$$\frac{dI(z)}{dz} = -(G+j\omega C)V(z)$$

$$I(z) = -\frac{1}{R+j\omega L}\frac{dV(z)}{dz}$$

$$\frac{d}{dz}\left(-\frac{1}{R+j\omega L}\frac{dV(z)}{dz}\right) = -(G+j\omega C)V(z)$$

$$\frac{d^2V(z)}{dz^2} = (R+j\omega L)(G+j\omega C)V(z)$$

$$\frac{d^2V(z)}{dz^2} - \left[\sqrt{(R+j\omega L)(G+j\omega C)}\right]^2 V(z) = 0$$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\begin{cases} \dfrac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \\ \dfrac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \end{cases} \qquad \gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$V(z) = \underbrace{V_0^+ e^{-\gamma z}}_{z+} + \underbrace{V_0^- e^{\gamma z}}_{z-}$$

$$\frac{dV(z)}{dz} = -(R+j\omega L)I(z)$$

$$\frac{d}{dz}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = -(R+j\omega L)I(z)$$

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -(R+j\omega L)I(z)$$

$$-\gamma\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = -(R+j\omega L)I(z)$$

$$I(z) = \frac{\gamma}{R+j\omega L}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right)$$

putem scrie: $I(z) = \frac{V(z)}{Z_0} \Rightarrow I(z) = \frac{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}{\frac{R+j\omega L}{\gamma}} = \frac{V(z)}{Z_0}$

Definim impedanța caracteristică a liniei

$$\boxed{Z_0 = \frac{R+j\omega L}{\gamma}}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_0^-| \cos(\omega t - \beta z + \phi^-) e^{-\alpha z}$$

$\phi^{\pm}$ - faza tensiunii complexe $V_0^{\pm}$

Parametrii undei:

$\lambda = \frac{2\pi}{\beta}$ — lungimea de undă pe linia de transmisie

$v_p = \frac{\omega}{\beta} = \lambda \cdot f \rightarrow$ viteza de propagare prin linia de transmisie.

P1.) $Z_0 = 75\ \Omega$

$i(t,z) = 1.8 \cos(3.77 \times 10^9 t - 18.13 z)$ mA

a) $f - ?$

b) $v_p = ?$

c) $\lambda = ?$

d) $\varepsilon_r = ?$

e) $I(z)$

f) $v(z,t)$

$I_0^+ = 1.88$ mA

$\omega = 3.77 \times 10^9 = 2\pi \cdot f \Rightarrow f = \frac{3.77 \times 10^9}{2\pi} = 0.6 \times 10^9$ Hz $= 600$ MHz.

$\beta = 18.13\ m^{-1}$

$v_p = \frac{\omega}{\beta} = \frac{3.77 \times 10^9}{18.13} = 0.208 \times 10^9$ m/s

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{18.13} = 0.346$ m $= 34.6$ cm.