Curs 4

(P.2.1) O linie de transmisie are $Z_0 = 75\,\Omega$, iar curentul care trece prin ea are forma:

$$i(t,z) = \underbrace{1.8}_{I_0^+} \cos(\underbrace{3.77\times10^9}_{\omega} t - \underbrace{18.13}_{\beta} z) \text{ mA}$$

a) $f = ?$

b) $v_p = ?$

c) $\lambda = ?$

d) $\varepsilon_r = ?$

e) $I(z) = ?$

f) $v(t,z) = ?$

a) $\omega = 2\pi f \Rightarrow f = \dfrac{\omega}{2\pi} = \dfrac{3.77\times10^9}{6.28} = 0.6\times10^9\text{ Hz} = 600\text{ MHz}.$

b) $v_p = \dfrac{\omega}{\beta} = \dfrac{3.77\times10^9\text{ s}^{-1}}{18.13\text{ m}^{-1}} = 0.2079\times10^9\text{ m/s} = 2.079\times10^8\text{ m/s}.$

c) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{18.13\text{ m}^{-1}} = 0.346\text{ m} = 34.6\text{ cm}$

d) $v_p = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} \Rightarrow v_p = \frac{1}{\sqrt{\varepsilon_r}} \cdot \underbrace{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}_{c} \Rightarrow$

$\mu = \mu_0$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\Rightarrow \frac{v_p}{c} = \frac{1}{\sqrt{\varepsilon_r}} \Rightarrow \frac{1}{\varepsilon_r} = \left(\frac{v_p}{c}\right)^2 \Rightarrow$

$\Rightarrow \varepsilon_r = \left(\frac{c}{v_p}\right)^2 = \left(\frac{2.998 \times 10^8 \ m/s}{2.079 \times 10^8 \ m/s}\right)^2 = 1.44^2 = 2.079 \simeq 2.08$

e) $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$

$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$\beta = 18.13 \ m^{-1}$

$\alpha = 0$

$\gamma = \alpha + j\beta$

$I_0^+ = 1.8 \ mA$

$I(z) = 1.8 \, e^{-18.13z} \ mA$

f) $z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} \Rightarrow V_0^+ = z_0 \cdot I_0^+ = 75 \ \Omega \times 1.8 \ mA =$

$= 135 \ mV = 0.135 \ V$

$V(z) = V_0^+ e^{-\gamma z} = 0.135 \cdot e^{-18.13z} \ V$

$v(z,t) = 0.135 \cos(3.77 \times 10^9 t - 18.13z) \ V$

Linia de transmisie cu pierderi mici

"low-loss transmission line"

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

$$Z_0 = \frac{R+j\omega L}{\gamma}$$

pierderi mici $\rightarrow$ $R \ll \omega L$ și $G \ll \omega C$ + frecv. înalte

$$\gamma = \sqrt{j\omega L\, j\omega C\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)} =$$

$$= j\omega\sqrt{LC}\sqrt{1+\frac{G}{j\omega C}+\frac{R}{j\omega L}+\underbrace{\frac{RG}{j^2\omega^2 LC}}_{\simeq 0}}$$

$$\gamma = j\omega\sqrt{LC}\,\underbrace{\sqrt{1-j\left(\frac{G}{\omega C}+\frac{R}{\omega L}\right)}}_{\sqrt{1+x}}$$

Dezvoltăm în serie Taylor: $\sqrt{1+x} = 1 + \frac{x}{2} + \dots$

$$\gamma \simeq j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] \simeq$$

$$\simeq j\omega\sqrt{LC} - \frac{j}{2}\cdot\frac{j\cancel{\omega}\sqrt{LC}\cdot R}{\cancel{\omega} L} - \frac{j}{2}\cdot\frac{j\cancel{\omega}\sqrt{LC}\cdot G}{\cancel{\omega} C} \simeq$$

$$\simeq j\omega\sqrt{LC} + \frac{1}{2}\left(R\cdot\sqrt{\frac{C}{L}} + G\cdot\sqrt{\frac{L}{C}}\right)$$

$$\gamma \simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \simeq \sqrt{\frac{L}{C}}$$

"high-freq., low-loss approx. for T. lines"

P.2.2.

$L = 0.5\,\mu H/m$ $\qquad f = 800\,MHz$

$C = 200\,pF/m$ $\qquad$ a) $\beta = ?$

$R = 4\,\Omega/m$ $\qquad$ b) $Z_0 = ?$

$G = 0.02\,S/m$ $\qquad$ c) atenuarea în dB dacă linia are o lungime de 30 cm

$$\beta = \omega\sqrt{LC} = 2\pi \times 800\times10^6 \sqrt{0.5\times10^{-6}\,H/m \times 200\times10^{-12}\,F/m} =$$

$$= 2\pi \times 800\times10^6 \sqrt{10^{-16}} = 2\pi \times 800 \times 10^6 \times 10^{-8} = 2\pi\cdot 8\ m^{-1} =$$

$$= 50.24\ m^{-1}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5\times10^{-6}\,H/\cancel{m}}{200\times10^{-12}\,F/\cancel{m}}} = \sqrt{\frac{0.5}{200}\times10^6} =$$

$$= \sqrt{2500} = 50\ \Omega$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) = \frac{1}{2}\left(\frac{4}{50} + 0.02\times50\right) = \frac{1}{2}(0.08+1) = \frac{1.08}{2} = 0.54.$$

$$V(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-j\beta z} e^{-\alpha z} \rightarrow A(z) - \text{amplitudinea (depinde de } z)$$

$$z = 0 \Rightarrow A(0) = V_0^+$$

$$z = 0.3 \Rightarrow A(0.3) = V_0^+ e^{-0.54\times0.3}$$

$$att\,(dB) = 20\log_{10}\frac{A(0.3)}{A(0)} = 20\log_{10}\frac{\cancel{V_0^+}e^{-0.162}}{\cancel{V_0^+}} =$$

$$= 20\times(-0.07) = -1.41\ dB$$

Linia de transmisie fără pierderi:

fără pierderi $\rightarrow R=0,\ G=0$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = j\,\omega\sqrt{LC} \Rightarrow \alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \frac{R+j\omega L}{\gamma} = \frac{\cancel{R}^{\,0}+\cancel{j}\omega L}{\cancel{j}\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$ - real

soluțiile generale pt. I și V:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

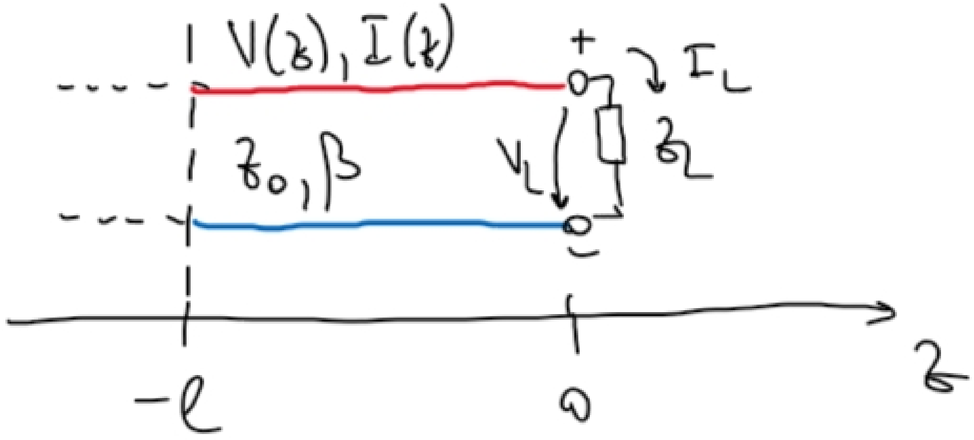
$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

$$\left.\begin{aligned} \lambda &= \frac{2\pi}{\beta} \\ \beta &= \omega\sqrt{LC} \end{aligned}\right\} \Rightarrow \lambda = \frac{2\pi}{\omega\sqrt{LC}}$$

$$v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}} = v_p$$

Cazuri particulare ale liniilor de transmisie fără pierderi:

a) Linia terminată în sarcină:



- sursa de semnal se află la $z<0$
- sursa generează un semnal de forma

$$V(z) = V_0^+ e^{-j\beta z}$$

$$Z_0 = \frac{V(z)}{I(z)} \qquad (*)$$

Dacă $Z_L = Z_0$ relația (*) este valabilă

Dacă $Z_L \neq Z_0$ → reflexie către generator + transmisie către Z_L
(mismatch)

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

unda incidentă

unda reflectată de sarcină

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

La sarcină ($z = 0$)

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} \Rightarrow V_0^- = \underbrace{\frac{Z_L - Z_0}{Z_L + Z_0}}_{\Gamma} V_0^+$$

Definim coeficientul de reflexie:

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\left.\begin{aligned} V(z) &= V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) \\ I(z) &= \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) \end{aligned}\right\}$$

suprapunere a 2 unde
$\Downarrow$
unde staționare

"Return Loss" (RL):

$$RL = -20\log|\Gamma| \quad [dB].$$

$\Gamma = 0 \Rightarrow RL = \infty$ dB (nu avem reflexie)

$|\Gamma| = 1 \Rightarrow RL = 0$ dB (întreaga putere incidentă e reflectată)
