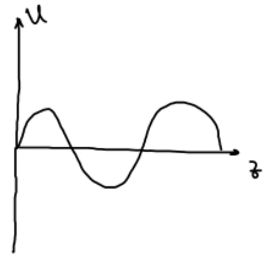
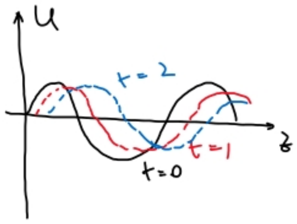
Curs 5:

$$\begin{cases} V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) \\ I(z) = \dfrac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) \end{cases}$$

Undă staționară




Undă care se propagă




$$z = a + j \cdot b = |z| e^{j\theta}$$
$$\theta = tg \frac{Im}{Re}$$

$$|V(z)| = \left|V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)\right|$$

$$|V(z)| = |V_0^+| \cdot \underbrace{|e^{-j\beta z}|}_{1} \cdot \left|1 + \Gamma e^{j\beta z} \cdot e^{j\beta z}\right| =$$

$$= |V_0^+| \left|1 + \Gamma e^{2j\beta z}\right|$$

$\Gamma = |\Gamma| e^{j\theta}$, $\theta$ - faza coeficientului de reflexie

(*) $|V(z)| = |V_0^+| \left|1 + |\Gamma| e^{j(\theta + 2j\beta z)}\right| \Rightarrow V$ oscilează în funcție de $z$.

$V_{max} = ?$
$V_{min} = ?$

$V_{max}$ se obține atunci când $e^{j(\theta + 2\beta z)} = 1$

$$V_{max} = |V_0^+| \, |1 + |\Gamma||$$

$V_{min}$ se obține atunci când $e^{j(\theta + 2\beta z)} = -1$

$$V_{min} = |V_0^+| \, |1 - |\Gamma||$$

Definim $SWR = \dfrac{V_{max}}{V_{min}} = \dfrac{|1 + |\Gamma||}{|1 - |\Gamma||} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

„Standing Wave Ratio"

$SWR = VSWR$ „Voltage Standing Wave Ratio"

$1 \le SWR \le \infty$

$SWR = 1 \rightarrow$ sarcină adaptată ($\Gamma = 0$).

(*) $\Rightarrow$ distanța dintre 2 maxime (sau minime) $z = \frac{2\pi}{2\beta} = \frac{\pi\lambda}{2\pi} = \frac{\lambda}{2}$

$$|V(z)| = |V_0^+|\,|1+|\Gamma|e^{j(\theta+2j\beta z)}|$$

$\lambda$ - lungimea de undă pe linia de transmisie

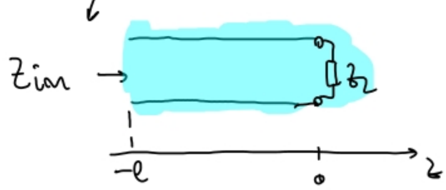
distanța dintre un maxim și un minim

$$z = \frac{\pi}{2\beta} = \frac{\lambda}{4}$$

Dacă $z = -l$

$$\Gamma(l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma(0)e^{-2j\beta l}$$

V, I variază în funcție de z $\Rightarrow$ impedanța văzută (măsurată) de la generator variază




$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{\cancel{V_0^+}(e^{j\beta l}+\Gamma e^{-j\beta l})}{\cancel{V_0^+}(e^{j\beta l}-\Gamma e^{-j\beta l})} Z_0 =$$

$$= \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}} Z_0$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Rightarrow$

$$\Rightarrow Z_{in} = Z_0 \frac{(Z_L+Z_0)e^{j\beta l} + (Z_L-Z_0)e^{-j\beta l}}{(Z_L+Z_0)e^{j\beta l} - (Z_L-Z_0)e^{-j\beta l}} =$$

$$= Z_0 \frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l\ jZ_L\sin\beta l}$$

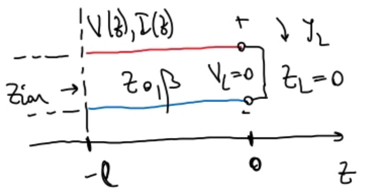
$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0\,\mathrm{tg}\,\beta l}{Z_0 + jZ_L\,\mathrm{tg}\,\beta l}}$$

- ecuația impedanței de intrare a liniei de transmisie

$l$ - lungimea liniei

$Z_L$ - impedanța de sarcină

# Linia terminată în scurtcircuit




$$Z_L = 0 \Rightarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1 \Rightarrow SWR = \infty$$

$$V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) = V_0^+ \left(e^{-j\beta z} - e^{j\beta z}\right) =$$

$$= -2j V_0^+ \sin \beta z$$

$$I(z) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} + e^{j\beta z}\right) =$$

$$= \frac{2V_0^+}{Z_0} \cos \beta z$$

La sarcină $(z = 0) \rightarrow V(0) = 0$

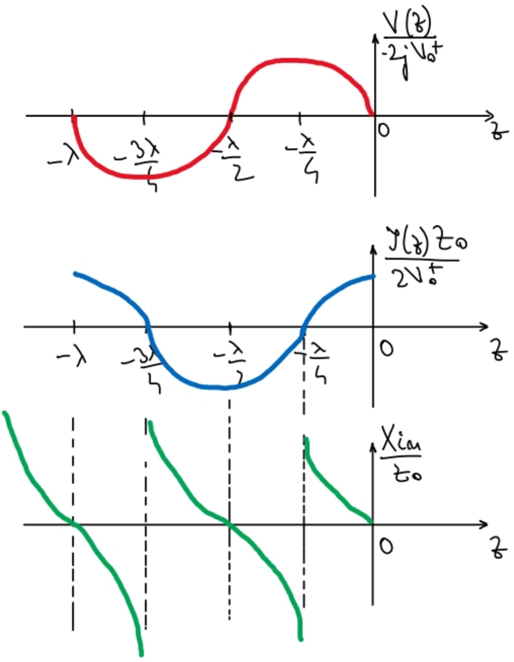
$I(0) \rightarrow$ maxim.

$$Z_{in} = j Z_0 \, tg \beta l \quad \text{sau} \quad \frac{V(-l)}{I(-l)}$$

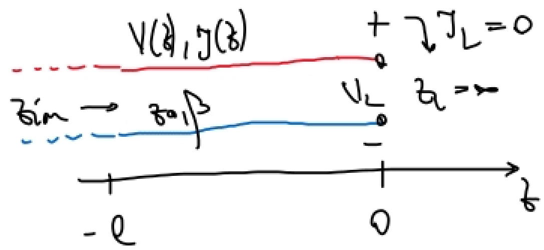
$Z_{in}$ - imaginar $\rightarrow -j\infty \rightarrow j\infty$

ex: dacă $l = 0 \Rightarrow Z_{in} = 0$

$l = \frac{\lambda}{4} \Rightarrow Z_{in} = \infty$



Linia de transmisie terminată în gol:




$$Z_L=\infty \Rightarrow \Gamma = 1 \Rightarrow SWR=\infty$$

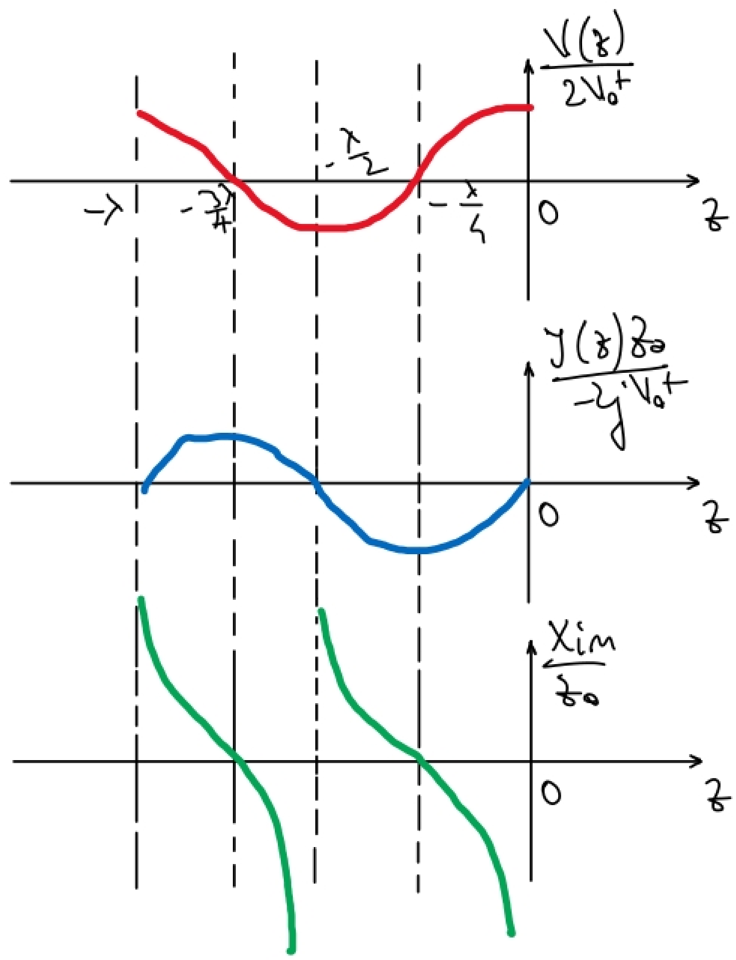
$$V(z) = V_0^+ \left(e^{-j\beta z} + e^{j\beta z}\right) = 2V_0^+ \cos\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - e^{j\beta z}\right) = -\frac{2jV_0^+}{Z_0}\sin\beta z$$

$I(0)=0$

$V(0) \to$ maxim

$$Z_{in} = -jZ_0 \operatorname{ctg}\beta l = \frac{V(-l)}{I(-l)}$$



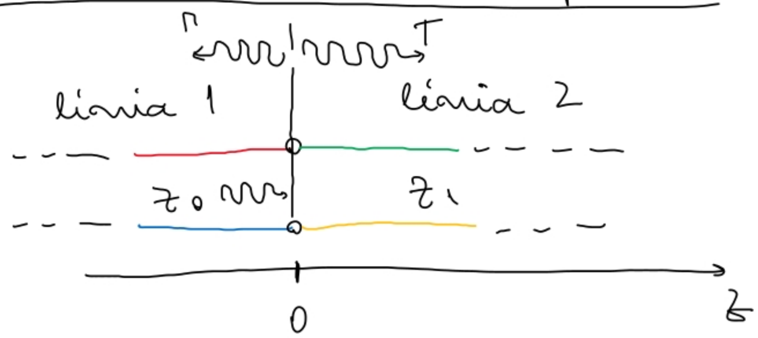
Linii cu lungimi speciale: (linii terminate în $Z_L$)

Dacă $\ell = \frac{\lambda}{2} \Rightarrow Z_{in} = Z_L \to$ indep. de $Z_0$

Dacă $\ell = \frac{\lambda}{4} + n\frac{\lambda}{2}$, $n = 1, 2, 3, \dots$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

## Reflexia și transmisia la joncțiunea a două linii de transmisie cu $Z_0$ diferite




$Z_0 \neq Z_1$

Liniile sunt infinit lungi (fără reflexii de la capăt)

La $z = 0$ $\quad \Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$ (reflexie + transmisie)

linia 1 $\qquad\qquad$ linia 2

$$\underbrace{\cancel{V_0^+}\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}_{V(0)} = \underbrace{\cancel{V_0^+}\, T e^{-j\beta z}}_{V(0)}$$

$z = 0 \Rightarrow T = 1 + \Gamma$

$$T = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$ coef. de transmisie

Definim "insertion loss" (IL):

$$\boxed{IL = -20 \log|T| \quad (dB)}$$

Decibeli

$$n = 10 \log \frac{P}{P_{ref}} \quad (dB)$$

$$dBm \rightarrow \quad n = 10 \log \frac{P}{1mW} \quad (dBm)$$

ex1 $\quad P = 2P_{ref} \Rightarrow n = 10 \log 2 = 10 \times 0.301 = +3dB$ (amplificare)

ex2 $\quad P = 0.1 P_{ref} \Rightarrow n = 10 \log 0.1 = -10\,dB$ (atenuare)

$$\sim \longrightarrow \boxed{+23dB} \longrightarrow \boxed{-6dB} \longrightarrow n$$

$$n = 23 - 6 = +17dB$$

Pentru tensiuni $\quad n = 20 \log \frac{V}{V_{ref}} \quad (dB)$

•) Neperi:

$$n = \ln \frac{V}{V_{ref}} \quad (Np)$$

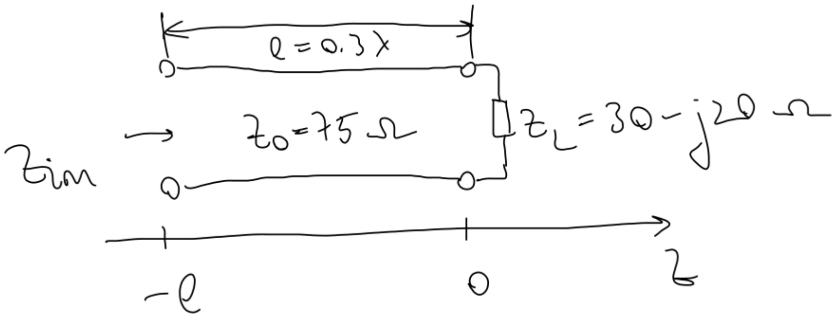
pt. puteri $\quad n = \frac{1}{2} \ln \frac{P}{P_{ref}}$

$$1\,Np \rightarrow \frac{P}{P_{ref}} = e^2$$

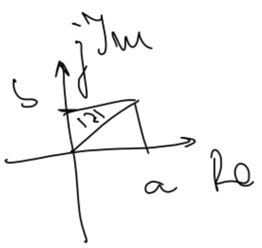
$$1\,Np = 10 \log e^2 = 8.686\,dB$$

**P2.8.** O linie de transmisie fără pierderi cu $\ell = 0.3\lambda$ terminată în $Z_L$.

$\Gamma(0) = ?$ ; $SWR = ?$ ; $\Gamma(-\ell) = ?$ ; $Z_{in} = ?$




$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20\,\Omega - 75}{30 - j20\,\Omega + 75} = \frac{-45 - j20}{105 - j20} =$$




$$= \frac{(-45 - j20)(105 + j20)}{(105 - j20)(105 + j20)} = \frac{-4725 - 2100j - 900j + 400}{11425} =$$

$$= -0.38 - 0.26j = 0.46 \cdot e^{j0.6} = 0.46 \angle 34.4^\circ$$

$$\theta = \operatorname{arctg} \frac{0.26}{0.38} = 0.6$$

$$|\Gamma| = \sqrt{0.38^2 + 0.26^2} = \sqrt{0.1444 + 0.0676} = 0.46$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.7$$

$$\Gamma(\ell) = \frac{V_0^- e^{-j\beta\ell}}{V_0^+ e^{j\beta\ell}} = \Gamma(0) e^{-2j\beta\ell}$$

$$\Gamma(-0.3\lambda) = \Gamma(0) e^{2j\beta \times 0.3\lambda} \quad \Gamma(0) e^{2j \cdot \frac{2\pi}{\cancel{\lambda}} \cdot 0.3\cancel{\lambda}} =$$

$$= \Gamma(0) e^{j \times 1.2\pi} = 0.46 \cdot e^{j(0.6 + 1.2\pi)} =$$

$$= 0.46 e^{j4.37}$$

$$\Gamma = |\Gamma| \angle \theta = 0.46 \angle 250.5^\circ$$