Examene ani terminali

Data 1: 27.05
Data 2: 02.06
Rest.: 08.06
ora 12

Date provizorii FT:

Data1: 23.06
Data2: 25.06
Rest:

Curs 11 SIS:

ZGOMOTE!

zgomot: orice semnal perturbator nedorit.

1. Zgomotul termic (Johnson-Nyquist):
- se datorează agitației termice a purtătorilor de sarcină într-un conductor

Densitatea de putere a zgomotului

$$\langle H \rangle = k_B T \quad [W/Hz]$$

Nivelul zgomotului termic pt. o bandă de frecvențe: (Δf)

$$N = k_B \cdot T \cdot \Delta f \quad [dBm]$$

$$N[dB] = 10 \log_{10} \frac{P}{P_{ref}}$$

$$N[dBm] = 10 \log_{10} \frac{P}{1mW}$$

La T=300K putem aproxima

$$\boxed{N_J\,[dBm] = -174 + 10 \log_{10} \Delta f}$$

- zgomotul termic → zgomot alb („white noise")
depinde doar de Δf dar nu și de poziția Δf în spectrul frecvențelor

pt. condensatori → zgomot „kTC"

$$v_n = \sqrt{\frac{k_B T}{C}}$$

Reducerea zgomotului termic:
→ răcire (L_{N_2} = 77K)
→ Δf cât mai îngust

Exemplu:

$R = 100\,k\Omega$

$T = 300\,K$

$\Delta f = 1\,kHz$

$v_{rms}^2 = 4 k_B T \cdot R \cdot \Delta f =$

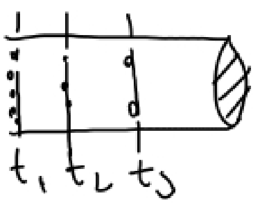
$= 4 \cdot 1.38 \cdot 10^{-23} \cdot 300 \cdot 100 \cdot 10^{3} =$

$= 16.56 \times 10^{-13}\,V^2$

$$\boxed{v_{rms} = 1.28\,\mu V}$$

2) Zgomot de alice („shot noise")

- electronii sunt particule discrete



→ într-un interval de timp infinitezimal nu trece aceeași sarcină

↓

fluctuații Poisson

$i_s^2 = \boxed{i_{rms}^2 = 2e \cdot I \cdot \Delta f}$ → zgomot alb

ex: 1.)

$e = 1.6 \times 10^{-19}\,C$

$\Delta f = 1\,kHz$

$I_{rms} = 10^{-9}\,A$

$\frac{i_s}{I_{rms}} \simeq 0.1\%$

$i_s^2 = i_{rms}^2 = 2 \times 1.6 \times 10^{-19} \times 10^{-9} \times 10^{3} =$

$= 0.32 \times 10^{-24}\,A^2$

$i_s = i_{rms} \simeq 0.57\,pA$

2.) $I_{rms} = 1\,mA$

$\Delta f = 1\,kHz$

$i_s = i_{rms} \simeq 0.57\,nA$

$\frac{i_s}{I_{rms}} \simeq 10^{-4}\%$

3.) $I_{rms} = 1\,A$

$\Delta f = 1\,kHz$

$i_s = 18\,nA$

$\frac{i_s}{I_{rms}} \simeq 10^{-7}\%$

3.) Zgomotul 1/f (flicker noise):

$$P \sim \frac{1}{f}$$ - zgomot roz

- depinde de calitatea circuitului/componentelor
- important pt. oscilatori RF.

4.) Zgomote externe (interferențe)

- câmpuri electromagnetice
- AC line 50Hz.

Raportul semnal-zgomot

„signal to noise ratio" (SNR)

$$SNR = \frac{P_{signal}}{P_{noise}} = \left(\frac{V_{rms}^{signal}}{V_{rms}^{noise}}\right)^2 = \left(\frac{I_{rms}^{signal}}{I_{rms}^{noise}}\right)^2$$

Îmbunătățirea SNR:

Metode HARDWARE:

- ecranare/pământare
- filtre active/pasive
- folosirea A.D. + A.I.
- modulația semnalelor
- lock-in

Metode SOFTWARE

- medierea
- filtrarea digitală

Proiectarea filtrelor. Transformata Laplace:

Probleme fizice, circuite → ecuații dif.
sisteme de ec. dif.
↳ soluții în t

Transformata Laplace:

t $\xrightarrow{\text{Laplace}}$ $s = \sigma + j\omega$ - variabilă complexă
$f(t)$ $F(s)$ → spațiul frecvențelor

$$\boxed{F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt}$$

$$\frac{df}{dt} \xrightarrow{\text{Laplace}} sF(s) - f(0)$$ → valoarea inițială în timp

$$\int f(t)dt \xrightarrow{\text{Laplace}} \frac{F(s)}{s} + \frac{1}{s}\left(\int_{-\infty}^{0^-} f(t)dt\right)$$

Considerăm inițial $i_L = 0$ și $v_C = 0$

•) Pt. un inductor:

Legea lui Ohm: $v(t) = L \cdot \frac{di}{dt} \longrightarrow V(s) = L \cdot s \cdot I(s)$

$$Z_L(s) = Ls$$

$$V(s) = Z_L \cdot I(s)$$

$$[Z_L] = H \cdot \frac{1}{s} = \frac{H}{s} = \frac{Wb}{As} = \frac{V\cancel{s}}{A\cancel{s}} = \frac{V}{A} = \Omega$$

•) Pt. un condensator:

$$v(t) = \frac{1}{C}\int i(t)dt \xrightarrow{\text{Laplace}} V(s) = \frac{I(s)}{Cs}$$

$$V(s) = \frac{1}{Cs} \cdot I(s)$$

$$V(s) = Z_C \cdot I(s)$$

$$Z_C = \frac{1}{Cs}$$

$$Z_C = \frac{1}{F Hz} = \frac{s}{F} = \frac{s \cdot V}{C} = \frac{V}{A} = \Omega$$

•) Pt. un rezistor:

$$v(t) = R \cdot i(t) \xrightarrow{\text{Laplace}} V(s) = R \cdot I(s)$$

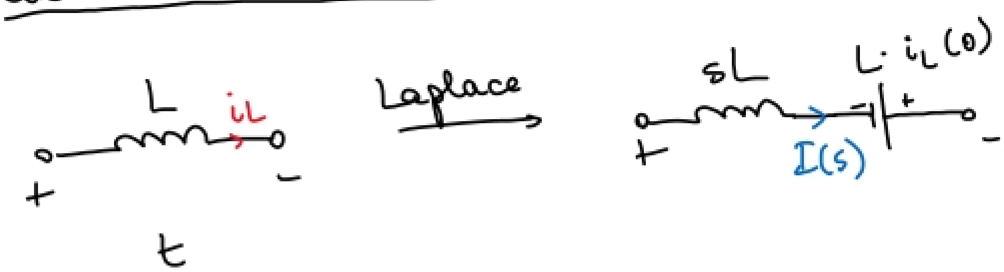
Mai general

•) Pt. un inductor:

$$v(t) = L \cdot \underbrace{\frac{di(t)}{dt}}_{s \cdot I(s) - i_L(0)} \xrightarrow{\text{Laplace}} V(s) = L \cdot s \cdot I(s) - L \cdot i_L(0)$$

tensiune constantă ce se opune (Lenz)

•) Circuit echivalent L:



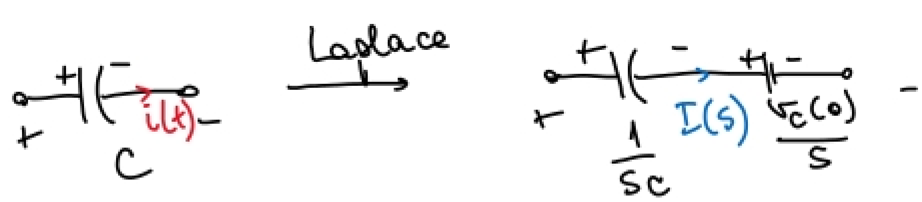
•) Pt. un condensator:

$$v(t) = \frac{1}{C} \int i(t)dt \xrightarrow{\text{Laplace}} V(s) = \frac{1}{sC} I(s) + \frac{1}{sC} \Big(\underbrace{\int_{-\infty}^{0} i_C(t)dt}_{q_C(0)} \Big)$$

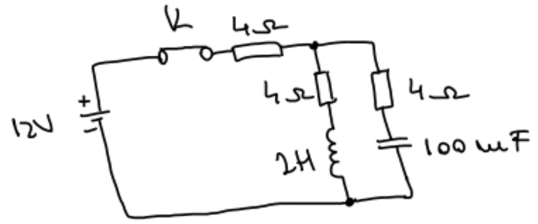
$$q_C(0) \;\text{ ||| }\; v_C(0)$$

$$V(s) = \frac{I(s)}{sC} + \frac{v_C(0)}{s}$$

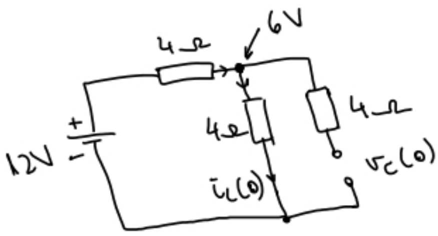
•) Circuit echivalent C:



P1. Determinați forma lui $i_L(t)$ după ce K este deschis:



circuitul înaintea deschiderii lui K $(t=0^-)$

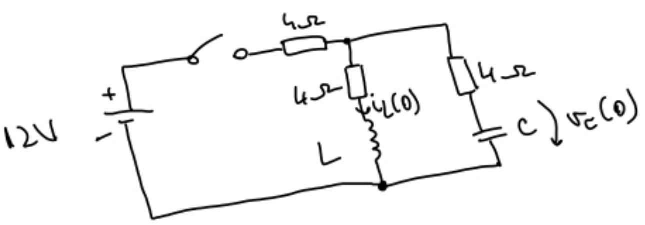


la $t=0^-$:

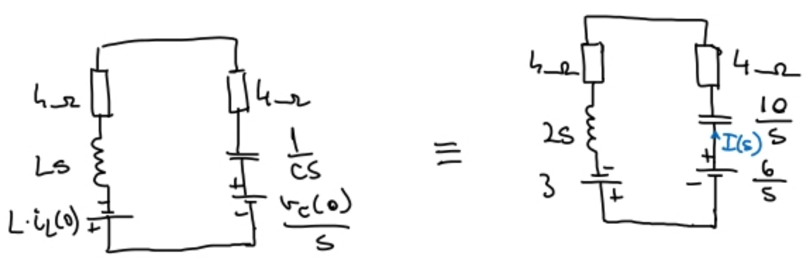
$$v_c(0)=6V$$

$$i_L(0)=\frac{12V}{4\Omega+4\Omega}=\frac{12}{8}=\frac{3}{2}=1.5A$$

Pentru $t=0$



Schema echivalentă Laplace pt. $t\geq 0$



$$\frac{6}{s}+3=I(s)\left(4+4+2s+\frac{10}{s}\right)$$

$$I(s)=\frac{\frac{6}{s}+3}{8+2s+\frac{10}{s}}=\frac{\frac{3s+6}{s}}{\frac{8s+2s^2+10}{s}}$$

$s=\sigma+j\omega$ [Hz]

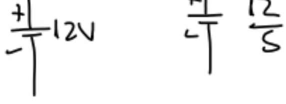
$$I(s)=I_L(s)=\frac{3s+6}{8s+2s^2+10}$$

Transf. Laplace Inversă $\boxed{i_L(t)=\frac{3}{2}e^{-2t}\cos t}$

$u(t)\longrightarrow\frac{1}{s}$

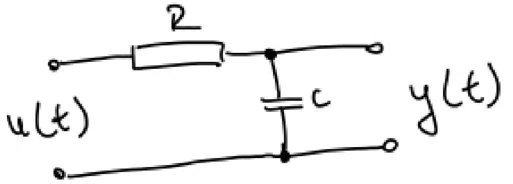
Surse DC $12V=12\cdot u(t)$

$u(t)$ - Heaviside step function



Filtre trece-jos (FTJ, LPF) unipol:

„single-pole filters".



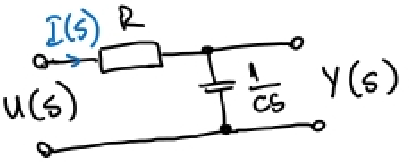
considerăm $v_c(0)=0$

$\downarrow$ Laplace

$$U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$H(s) = \frac{Y(s)}{U(s)}$$ – funcția de transfer

circuitul echivalent Laplace:



$$Y(s) = I(s)\cdot\frac{1}{cs}$$

$$U(s) = I(s)\left(R+\frac{1}{cs}\right)$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{cs}}{R+\frac{1}{cs}} = \frac{1}{cs}\cdot\frac{1}{R+\frac{1}{cs}}$$

$$\boxed{H(s) = \frac{1}{1+RCs}}$$ – raport de 2 polinoame

filtrul are 1 pol

$$H(s) = \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

z_i – zerouri

p_i – poli

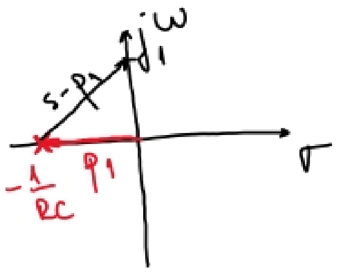
$$1+RCs = 0$$

$$RCs = -1$$

$$s = -\frac{1}{RC}$$

$\sigma + j\omega = -\frac{1}{RC}$ ⇒ filtrul are un pol la $\sigma = -\frac{1}{RC}$

(sistem stabil pt. $\sigma < 0$)



$$|H(s)| = \frac{1}{|1+RC(\sigma+j\omega)|} = \frac{1}{\sqrt{(1+RC\sigma)^2+(RC\omega)^2}}$$

$R = 100\ \Omega$

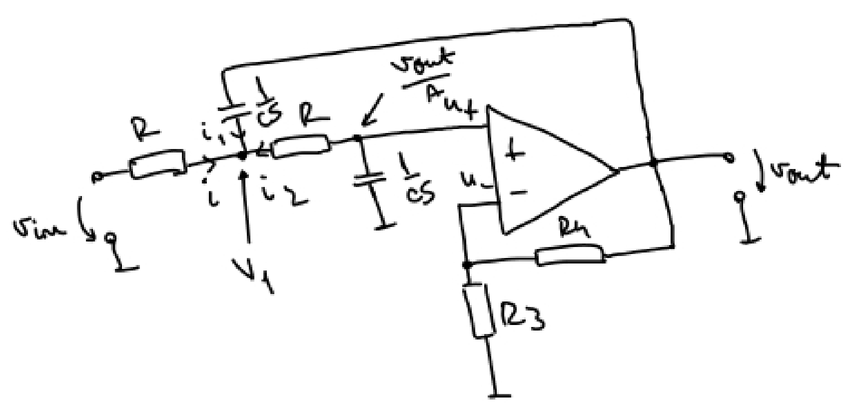
$C = 470\ \mu F$

$$|H(s)| = \frac{1}{\sqrt{(1+\sigma\cdot 47\cdot 10^{-3})^2+(\omega\cdot 47\cdot 10^{-3})^2}}$$

$\omega, \sigma \to 10^6$

$$|H(s)| = \frac{1}{\sqrt{(1+47\sigma)^2+(47\omega)^2}}$$

Filtru Trece-Jos Sallen-Key:



$u_+ \cong u_-$

conexiune neinversoare $A = 1 + \frac{R_4}{R_3}$

$$u_- = \frac{R_3}{R_3+R_4} v_{out} = v_{out}\cdot\frac{1}{1+\frac{R_4}{R_3}} = \frac{v_{out}}{A}$$

$$V_1 = I_2\cdot R + \frac{V_{out}}{A}$$

$$\frac{I_2}{CS} = \frac{V_{out}}{A} \Rightarrow I_2 = \frac{V_{out}}{A}\cdot CS$$

$$V_1 = \frac{V_{out}}{A}\cdot RCS + \frac{V_{out}}{A} = \frac{V_{out}}{A}(1+RCS)$$

$$I_2 = I_1 + I$$

$$\frac{I_1}{CS} = V_{out} - V_1 \Rightarrow I_1 = (V_{out} - V_1)\,CS$$

$$IR = V_{in} - V_1 \Rightarrow I = \frac{V_{in} - V_1}{R}$$

$$\frac{V_{out}}{A} CS = (V_{out} - V_1)\,CS + \frac{V_{in} - V_1}{R} \quad \Big|\, AR$$

$$\left.\begin{aligned} V_{out} RCS &= (V_{out} - V_1) ARCS + A(V_{in} - V_1) \\ V_1 &= \frac{V_{out}}{A}(1 + RCS) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow V_{out} RCS = V_{out} ARCS - V_{out} RCS(1 + RCS) +$$
$$+ A \cdot V_{in} - \cancel{A} \cdot \frac{V_{out}}{\cancel{A}}(1 + RCS)$$

$$V_{out}\left[RCS - ARCS + RCS(1 + RCS) + 1 + RCS\right] = AV_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + 2RCS - ARCS + RCS + R^2C^2s^2}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{s^2R^2C^2 + sRC(3 - A) + 1} = H(s)$$

- filtrul are 2 poli

$$\Delta = R^2C^2(3 - A)^2 - 4R^2C^2$$

$$s_{1,2} = \frac{RC(A - 3) \pm \sqrt{R^2C^2(3 - A)^2 - 4R^2C^2}}{2R^2C^2}$$

$$= \frac{A - 3}{2RC} \pm \frac{\sqrt{(3 - A)^2 - 4}}{2RC}$$

$$s_{1,2} = \frac{A - 3}{2RC} \pm j\,\frac{\sqrt{-A^2 + 6A - 5}}{2RC}$$

Pentru simplitate alegem $R=1\,\Omega$, $C=1\,F$

pt. $A=1$

$$s_{1,2} = -\frac{1}{RC} \pm j \cdot \frac{\sqrt{-1+\cancel{6}-5}^{\;\to 0}}{\cancel{2RC}}$$

$$H(s) = \frac{1}{(s+1)(s+1)}$$

$$s_{1,2} = -1$$

$$|H(s)| = \frac{1}{|(s+1)(s+1)|}$$

$$|H(s)| = \frac{1}{|\sigma+j\omega+1|\,|\sigma+j\omega+1|} = \frac{1}{\sqrt{(\sigma+1)^2+\omega^2}\sqrt{(\sigma+1)^2+\omega^2}}$$

$\sigma = 0,\ \omega \geq 0 \Rightarrow$

$$\Rightarrow |H(s)| = \frac{1}{\sqrt{1+\omega^2}} \cdot \frac{1}{\sqrt{1+\omega^2}} = \frac{1}{1+\omega^2}$$

pt. $A=1.586$ $\quad s_{1,2} = -0.707 \pm j \cdot 0.707$

$$H(s) = \frac{1}{(s+0.707-j\,0.707)(s+0.707+j\,0.707)} =$$

$$= \frac{1}{[\sigma+0.707+j(\omega-0.707)][\sigma+0.707+j(\omega+0.707)]}$$

$$|H(s)| = \frac{1}{\sqrt{(\sigma+0.707)^2+(\omega-0.707)^2}\sqrt{(\sigma+0.707)^2+(\omega+0.707)^2}}$$

pt. $A=2.5$ $\quad s_{1,2} = -0.25 \pm j\,0.97$