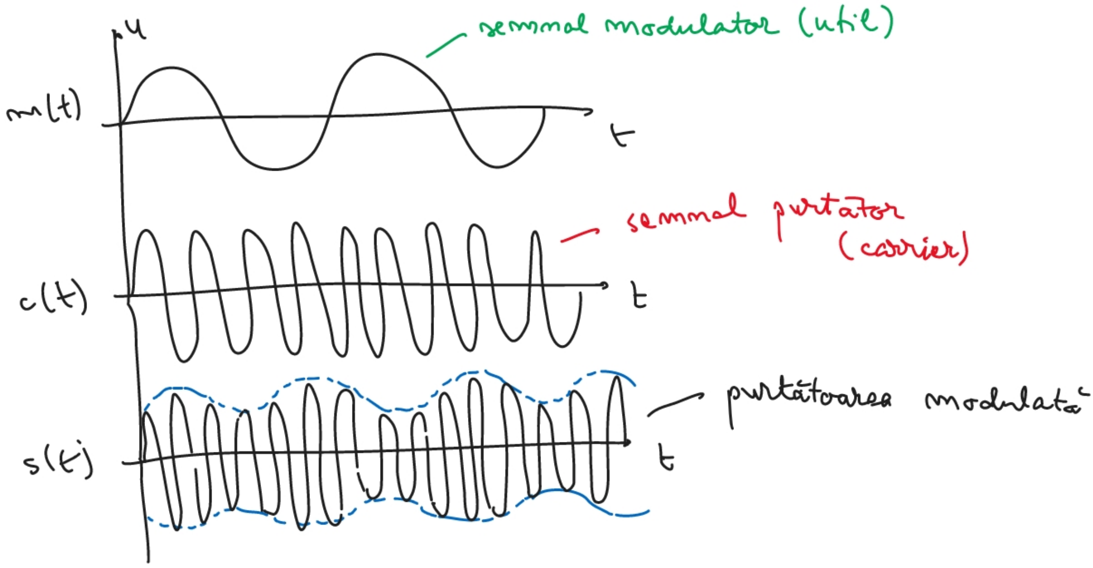
Curs 13 SiS:

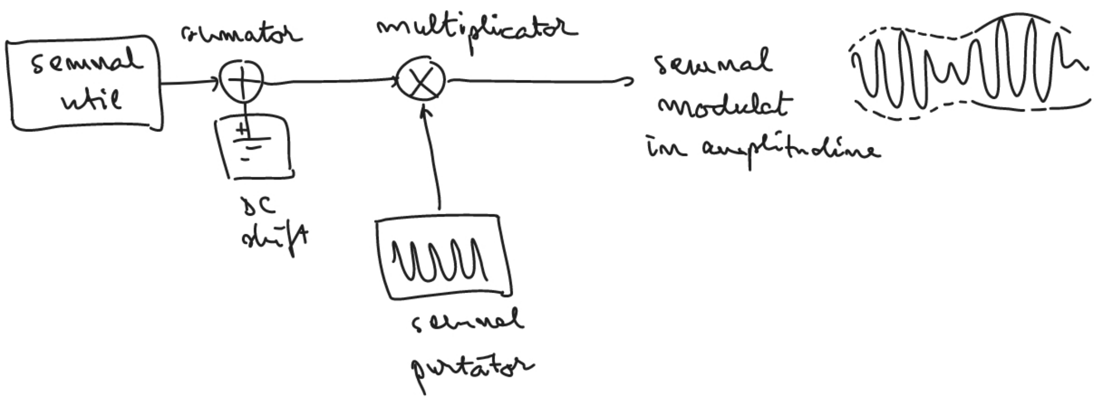
Modularea semnalelor

Modulație → analogică (AM, $\underbrace{\text{FM, PM}}_{\text{angle modulation}}$)
→ digitală (ASK, FSK, PSK (BPSK), QAM)

Modulația în amplitudine: (AM)



Schemă bloc pt. modulator AM:



semnal modulator: $m(t) = A_m \cos \omega_m t$

semnal purtător: $c(t) = A_c \cos \omega_c t$

semnalul modulat: $s(t) = [A_c + A_m \cos(\omega_m t)] \cos(\omega_c t)$

$$s(t) = A_c \left[1 + \underbrace{\frac{A_m}{A_c}}_{\mu} \cos \omega_m t\right] \cos(\omega_c t)$$

$$s(t) = A_c [1 + \mu \cos \omega_m t] \cos \omega_c t$$

$\mu = \frac{A_m}{A_c}$ - factorul de modulație

A_{max} - amplit. max. a lui $s(t)$

A_{min} - amplit. min. a lui $s(t)$

$$A_{max} \to \cos \omega_m t = 1 \Rightarrow A_{max} = A_c + A_m$$

$$A_{min} \to \cos \omega_m t = -1 \Rightarrow A_{min} = A_c - A_m$$

$$A_{max} + A_{min} = A_c + \cancel{A_m} + A_c - \cancel{A_m}$$

$$A_c = \frac{A_{max} + A_{min}}{2}$$

$$A_{max} - A_{min} = \cancel{A_c} + A_m - \cancel{A_c} + A_m$$

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$\mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$\mu = 1 \to$ modulație perfectă

$\mu < 1 \to$ undă submodulată („undermodulated")

$\mu > 1 \to$ undă supramodulată („overmodulated")

·) Lărgimea de bandă:

$$BW = f_{max} - f_{min} \ [Hz]$$

$$s(t) = A_c[1 + \mu \cos(\omega_m t)]\cos(\omega_c t) =$$

$$= A_c \cos(\omega_c t) + \mu A_c \cos(\omega_c t)\cos(\omega_m t) =$$

$$\left[\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}\right]$$

$$s(t) = A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos[(\omega_c + \omega_m)t] + \frac{\mu A_c}{2}\cos[(\omega_c - \omega_m)t]$$

3 frecvențe: f_c, $\underbrace{f_c + f_m}_{BLS}$, $\underbrace{f_c - f_m}_{BLI}$

USB („upper sideband") — LSB („lower sideband")

$$BW = \underbrace{\cancel{f_c} + f_m}_{USB} - \underbrace{\cancel{f_c} + f_m}_{LSB} = 2 f_m$$

i) Puterea semnalului modulat AM:

$$P_T = P_C + P_{USB} + P_{LSB}$$

$$\left[P_{RMS} = \frac{V_{RMS}^2}{R} = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{R} \right.$$

$$\left. V = A\cos\omega t \right]$$

$$P_C = \frac{A_C^2}{2R}$$

$$P_{USB} = \frac{\mu^2 A_C^2}{8R}$$

$$P_{LSB} = \frac{\mu^2 A_C^2}{8R}$$

$$P_T = \frac{A_C^2}{2R} + 2\frac{\mu^2 A_C^2}{8R}$$

$$P_T = \underbrace{\frac{A_C^2}{2R}}_{P_C}\left(1 + \frac{\mu^2}{2}\right)$$

$$P_T = P_C\left(1 + \frac{\mu^2}{2}\right)$$

P1.

$$m(t) = 10\cos(2\pi \cdot 10^3 t)$$
$$c(t) = 50\cos(2\pi \cdot 10^5 T)$$

Determinați μ, P_C, P_T $(R = Z_0 = 50\ \Omega)$

$f_m = 1$ kHz $\qquad A_m = 10$ V

$f_C = 100$ kHz $\qquad A_C = 50$ V

$$\mu = \frac{A_m}{A_C} = \frac{1}{5} = 0.2 \quad \text{(20\% modulație)}$$

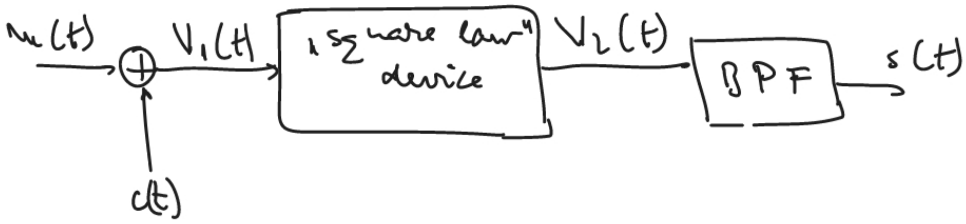
$$P_C = \frac{A_C^2}{2R} = \frac{50^2}{100} = 25\ W$$

$$P_T = P_C\left(1 + \frac{\mu^2}{2}\right) = P_C\left(1 + \frac{4 \times 10^{-2}}{2}\right) =$$

$$= P_C \times 1.02 = 25.5\ W$$

Modulatoare AM
- „square law"
- în comutație (switching)

·) Modulator „square law":



$$V_1(t) = m(t) + A_c \cos(w_c t)$$

$$V_2(t) = k_1 V_1(t) + k_2 V_1^2(t)$$

k_1, k_2 - constante

$$V_2(t) = k_1 [m(t) + A_c \cos(w_c t)] + k_2 [m(t) + A_c \cos(w_c t)]^2 =$$

$$= k_1 m(t) + k_1 A_c \cos(w_c t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(w_c t) +$$
$$+ 2 k_2 m(t) A_c \cos(w_c t)$$

$$V_2(t) = k_1 m(t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(w_c t) +$$
$$+ k_1 A_c \left[1 + m(t) \frac{2k_2}{k_1}\right]$$

$$s(t) = k_1 A_c [1 + k_a m(t)] \cos(w_c t)$$

k_1 - factor de scală

k_a - sensibilitatea în amplitudine

·) modulator în comutație:

$$V_1(t) = m(t) A_c \cos(w_c t)$$

pp. că $A_m << A_c$

$$V_2(t) = \begin{cases} V_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

$$V(t) = V_1(t) \cdot x(t)$$

Dezv. $x(t)$ în serie Fourier:

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \dots$$

$$V_2 = [m(t) + A_c \cos \omega_c t]\left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \dots\right]$$

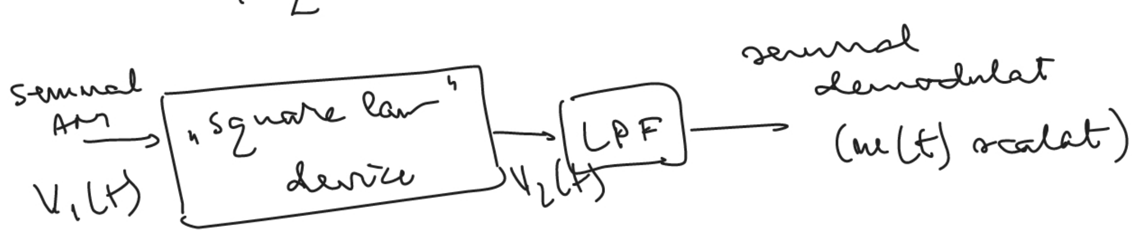
$$V_2 = \underbrace{\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos(\omega_c t)}_{\uparrow\ \text{BPF}} + \frac{m(t)}{2} + \frac{2A_c}{\pi}\cos^2(\omega_c t) + \dots$$

$$s(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos(\omega_c t)$$

$$k_1 = 0.5$$

$$k_a = \frac{4}{\pi A_c}$$

·) Demodulator „square law":



$$V_1(t) = A_c(1 + k_a m(t)) \cos(\omega_c t)$$

$$V_2(t) = k_1 V_1(t) + k_2 V_1^2(t) =$$

$$= k_1 A_c[1 + k_a m(t)] \cos(\omega_c t) +$$

$$+ k_2 A_c^2 [1 + k_a m(t)]^2 \cos^2(\omega_c t)$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$v_2(t) = k_1 A_c [1 + k_a m(t)]^2 \cos(w_c t) + \\ + k_2 A_c^2 (1 + k_a m(t))^2 \frac{1 + \cos(2 w_c t)}{2}$$

$$v_2(t) = k_1 A_c [1 + k_a m(t)] \cos w_c t) + \\ + \underbrace{\frac{k_1 A_c^2}{2}}_{DC} + \frac{k_2 k_a^2 A_c^2 m(t)}{2} + \underbrace{k_2 k_a A_c^2 m(t)}_{\text{semnal util}} +$$

$$+ k_2 A_c^2 [1 + k_a m(t)]^2 \frac{\cos(2 w_c t)}{2}$$

După LPF

$$v_2(t) = \frac{k_2 A_c^2}{2} + k_2 k_a A_c^2 m(t)$$

După condensator de cuplaj

$$v_2(t) = k_2 k_a A_c^2 m(t)$$

Modulația în frecvență (FM)

undă modulată unghiular

$$s(t) = \underset{\text{const.}}{A_c} \cos(\theta_i(t))$$

$$f_i = f_c + k_f m(t)$$

k_f – sensib. în frecv. [Hz/V]

$$w_i = \frac{d\theta_i}{dt}$$

$$2\pi f_i = \frac{d\theta_i(t)}{dt} \Rightarrow \theta_i(t) = 2\pi \int f_i dt$$

$$\theta_i(t) = 2\pi \int [f_c + k_f m(t)] dt$$

$$s(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)]$$

Pt. FM $\Rightarrow m(t) = A_m \cos(2\pi f_m t)$

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]}$$

$$\beta = \frac{k_f \cdot A_m}{f_m} = \frac{\Delta f}{f_m}$$ – fact. de modulație

Δf – deviația în frecvență

$\beta < 1 \to$ narrow band FM (NBFM) $\to B_W \simeq 2 f_m$

$\beta > 1 \to$ wideband FM (WBFM) $\to B_W = \Delta f = 2(\beta+1) f_m$

Modulația în fază:

$$\phi_i = k_p \cdot m(t)$$

k_p – sensib. în fază

$$s(t) = A_c \cos(2\pi f_c + \phi_i)$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$s(t) = A_c \cos[2\pi f_c t + \beta \cos(2\pi f_m t)]$$

$\beta = \Delta\phi = k_p A_m$ – fact. de modulație

$\Delta\phi$ – deviația în fază.